Journal of Environmental Statistics

February 2010, Volume 1, Issue 3. http://www.jenvstat.org

Exponentiated Gumbel Distribution for Estimation of Return Levels of Significant Wave Height

Klara Persson
Skandia, Stockholm

Jesper Rydén
Department of Mathematics
Uppsala University

Abstract

The exponentiated Gumbel (EG) distribution has been proposed as a generalization of the classical Gumbel distribution. In this paper we discuss estimation of T-year return values for significant wave height in a case study and compare point estimates and their uncertainties to the results given by alternative approaches using Gumbel or Generalized Extreme Value distributions. A jackknife approach is made to investigate the sensitivity of the parameter estimates and various model selection criteria are employed to compare the models. When examining Anderson–Darling distances between samples and extreme-value distribtions, the EG distribution turns out to give the closest fit. However, general recommendations whether to use Gumbel or EG distribution cannot be given.

Keywords: Gumbel distribution, exponentiated distributions, return value, significant wave height, model selection.

1. Introduction

A frequently occurring problem in statistics is model selection and related issues. In standard applications like regression analysis, model selection may be related to the number of independent variables to include in a final model. In some applications of statistical extreme-value analysis, convergence to some standard extreme-value distributions is crucial. A choice has occasionally to be made between special cases of distributions versus the more general versions. In this paper, statistical properties of a recently proposed distribution is examined closer and a case study is performed where comparison is made to classical distributions.

In applications of extreme-value analysis to risk analysis, computation of return levels are of importance, often for some quantity obtained from environmental data (wind speeds, wave heights, maximum rainfall). The 100-year return level is a value which is exceeded in average only once per 100 years. Usually this is estimated as a quantile of some extreme-value distribution, as will be defined in Section 3.2 (see Chapter 10 in Rychlik and Rydén (2006) for elementary discussion). The uncertainty of the obtained point estimate of the return level is of interest and might be considerable; hence related confidence intervals are of interest. In extreme-value analysis, it is shown that the maximum of many of the common distributions (normal, lognormal) converges to the Gumbel distribution,

$$F(x) = \exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}, \quad -\infty < x < \infty,$$

where $\sigma > 0$, $-\infty < \mu < \infty$. This is a special case of the so-called Generalized Extreme Value (GEV) distribution and in the literature of probabilistic risk analysis, there has been discussion about the choice of Gumbel compared to GEV. To quote Coles and Pericchi (2003), where analysis of rainfall measurements was made: “Although standard tests may support a reduction to the Gumbel family, this is a risky strategy.” Similar conclusions are given by Koutsoyiannis and Baloutsos (2000). The Gumbel distribution yields narrower confidence intervals than the three-parameter GEV but has also the risk of under-estimating the return level. Hence, the choice of distribution is not trivial.

Recently, a generalization of the Gumbel distribution, called the exponentiated Gumbel (EG) distribution, was introduced (Nadarajah, 2006):

$$F_{\mathrm{EG}}(x) = 1 - \left[1 - \exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]^{\alpha}, \quad -\infty < x < \infty, \tag{1}$$

where $\alpha > 0$, $\sigma > 0$. Moreover, hazard-rate functions, moments, asymptotics and maximum-likelihood functions were presented. A numerical illustration was given; computation of return levels for one single data set of annual maximum daily rainfall. A comparison was made with the Gumbel and GEV distributions where the EG distribution proved to be advantageous.

In this article, we further extend the analysis of the EG. Based on real data sets, return levels are computed. The resulting point estimates are compared with results based on Gumbel and GEV by performing a model selection based on deviance. As the question of uncertainty of estimates is important, we give related approximate confidence intervals for the return levels. The datasets are observations of significant wave height and originate from two buoys in the Pacific Ocean. The data was chosen because of the authors’ interest in and experience of modelling such quantities, cf. e.g. Rychlik, Rydén and Anderson (2009), where new estimation methodologies are presented based on theory for crossings in stationary Gaussian processes. Measurements of this type have often been collected for a time period of the order of some decades; hence, when regarding annual maxima, relatively small samples result.

The paper is organized as follows: in the next section, we give a brief orientation on exponentiated distributions. In Section 3, estimation of T-year return values is discussed and expressions for confidence intervals by the delta method are explicitly stated. The remaining sections are devoted to data analysis: in Section 4, two data sets are fitted to the EG distribution and we discuss whether the fit is reasonable. Moreover, in Section 5, comparison of the results for T-year return values using EG, GEV and Gumbel distributions is made, using various model selection criteria, such as deviance and comparison of Anderson Darling distances.

2. On exponentiated distributions

During the first half of the nineteenth century, certain cumulative distributions were introduced by B. Gompertz and P.F. Verhulst, for instance

$$F(x;\,\theta,\rho,\sigma) = (1-\rho \mathrm{e}^{-x/\sigma})^{\theta}, \quad x > \sigma \ln \rho,$$

see Ahuja and Nash (1967). More recently, the exponentiated exponential family, with distribution function

$$F(x;\,\alpha,\lambda) = (1-\mathrm{e}^{-\lambda x})^{\alpha}, \quad \alpha,\lambda > 0, \quad x > 0,$$

has been extensively studied in a number of papers by Gupta and Kundu, see for instance Gupta and Kundu (2001). The families of exponential and Weibull distributions are found within the exponentiated exponential distribution and therefore studies were performed to investigate asymptotic results as well as fits to data sets. More generally, distributions $F(x) = [G(x)]^{\alpha}$ where $G(x)$ is a distribution family and $\alpha > 0$ are occasionally called Lehmann alternatives in the context of modelling of failure times (Gupta, Gupta and Gupta 1998). However, note that the definition in Eq. (1) is rather of the form $F_{\mathrm{EG}}(x) = 1 - [1 - G(x)]^{\alpha}$, where $G(x)$ is the distribution function of the Gumbel distribution. In this paper, we have kept this definition, as it was stated by Nadarajah (2006). A reparametrization is possible.

Turning to extreme-value distributions, various forms of generalized extreme-value distributions have been proposed in the literature, e.g. a four-parameter distribution by Scarf (1992). For a review see Kotz and Nadarajah (2000), Chapter 2.7. We find it interesting to compare the three-parameter EG distribution to another three-parameter family — the GEV distribution — as well as the Gumbel distribution (EG with $\alpha = 1$). The general class of exponentiated distributions is closed under maximum (Gupta and Kundu 2007), that is, if $X_1, \ldots, X_n$ are iid random variables then the X_i variables are exponentiated random variables if and only if the maximum of $X_1, \ldots, X_n$ is an exponentiated random variable.

3. Extreme-value modelling

In this section we first give a brief review on classical extreme-value modelling and then present the estimation framework for the exponentiated Gumbel distribution. For further reference, consult Coles (2001) or Rychlik and Rydén (2006).

Suppose $X_1, \ldots, X_n$ is a sequence of independent and identically distributed variables, and let $M_n = \max\{X_1, \ldots, X_n\}$. Classical extreme-value theory is concerned with the limiting distribution of M_n as $n \to \infty$, or rather its normalized version: If there exist sequences of constants $\{a_n > 0\}$ and $\{b_n\}$ such that

$$\mathsf{P}((M_n - b_n)/a_n \le x) \to F(x) \quad \text{as} \quad n \to \infty,$$

the Extremal Types Theorem states that G must belong to one of three families of distributions (Gumbel, Frechet and Weibull). These can combined into a single family, the Generalized Extreme Value (GEV) distribution

$$F(x) = \begin{cases} \exp(-(1-\xi(x-\mu)/\sigma)^{1/\xi}), & \xi \neq 0, \\ \exp(-\exp(-(x-\mu)/\sigma)), & \xi = 0, \end{cases}$$

for $x > \sigma/\xi + m$ (when $\xi \leq 0$) and $x < \sigma/\xi + m$ (when $\xi > 0$). As the special case $\xi = 0$ is found the Gumbel distribution.

A common procedure in applications to environmental data is the method of block maxima. The original sequence is broken up into blocks of size n, say, and the maximum observation is extracted from each block. For the resulting sequence of iid observations, an extreme-value distribution is fitted. Often in applications the block size is chosen to be one year, and the goal is to estimate quantiles of the distribution of block maxima, so-called return levels. The method of block maxima is robust and used in codes but lots of data are disregarded. Other approaches exist, for instance threshold methods.

Often measurements are made at networks of stations, and a statistical problem is then to pool the information. It is out of the scope of this paper though to discuss such procedures; we focus at data collected at one individual setting at a time and then use the classical approach of block maxima.

3.1. ML estimation of exponentiated Gumbel distribution

In this section we present formulae related to estimation in the EG distribution. Asymptotic results and aspects on estimation were given in Nadarajah (2006). We do not explicitly give the corresponding formulae for the Gumbel distribution or GEV distribution although these are used later in the analysis of the data sets but refer to textbooks on extreme-value analysis and its applications, e.g. Rychlik and Rydén (2006), Chapter 10.

In the sequel we consider estimation by the method of maximum likelihood (ML). However, note that specialized procedures have been developed in the literature for estimation of parameters in extreme-value distributions (Coles and Dixon, 1999). The log-likelihood for a random sample $x_1, \ldots, x_n$ from the exponentiated Gumbel distribution is found in Nadarajah (2006) and we present it again here for the reader's convenience:

$$\begin{aligned}\log L(\alpha,\sigma,\mu) = n\log\alpha - n\sigma + (\alpha-1)\sum_{i=1}^{n}\log(1-\mathrm{e}^{-\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}}) \\ -\sum_{i=1}^{n}\frac{x_i-\mu}{\sigma} - \sum_{i=1}^{n}\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}.\end{aligned}$$

Moreover, the first-order derivatives of $l(\alpha,\sigma,\mu) = \log L(\alpha,\sigma,\mu)$ with respect to the three parameters are:

$$\begin{aligned}\frac{\partial l}{\partial \alpha} &= \frac{n}{\alpha} + \sum_{i=1}^{n}\log(1-\mathrm{e}^{-\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}}) \\ \frac{\partial l}{\partial \sigma} &= -\frac{n}{\sigma}\sum_{i=1}^{n}\frac{x_i-\mu}{\sigma^2}(1-\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}) + \frac{\alpha-1}{\sigma^2}\sum_{i=1}^{n}\frac{(x_i-\mu)\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}\mathrm{e}^{-\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}}}{1-\mathrm{e}^{-\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}}}, \\ \frac{\partial l}{\partial \mu} &= \frac{n}{\sigma} - \frac{1}{\sigma}\sum_{i=1}^{n}\mathrm{e}^{-\frac{x_i-\mu}{\sigma}} + \frac{\alpha-1}{\sigma}\sum_{i=1}^{n}\frac{\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}\mathrm{e}^{-\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}}}{1-\mathrm{e}^{-\mathrm{e}^{-\frac{x_i-\mu}{\sigma}}}}.\end{aligned}$$

By using approximate normality of ML estimates, the so-called delta method can be applied to construct approximate confidence intervals for functions of the estimated parameters, in our case, the T-year return level. (Note that in the literature on statistical extreme-value analysis, intervals based on profile likelihood are usually preferred (see e.g. Coles and Dixon 1999) and bootstrap approaches have also been suggested. We find it though out of the scope of this article to evaluate different methods for obtaining confidence intervals.)

3.2. T-year return values and related intervals

In hydrology, oceanography and other fields of application, the T-year return level x_T is defined through the relation $F(x_T) = 1 - 1/T$ and for the EG distribution it follows that an estimate is given by

$$\hat{x}_T = \hat{\mu} - \hat{\sigma} \ln(-\ln(1 - (1/T)^{1/\hat{\alpha}})).$$

For the convenience of the reader and possible future implementations, we give some details of the confidence interval for the return level as obtained by the delta method:

$$(\hat{x}_T - \lambda_{\alpha/2}\hat{D} \leq x_T \leq \hat{x}_T + \lambda_{\alpha/2}\hat{D}).$$

Here

$$\hat{D}^2 = \nabla x_T(\hat{\alpha}, \hat{\sigma}, \hat{\mu})^{\mathsf{T}} \widehat{\boldsymbol{\Sigma}}\, \nabla x_T(\hat{\alpha}, \hat{\sigma}, \hat{\mu}),$$

where the gradient vector is

$$\nabla x_T(\hat{\alpha}, \hat{\sigma}, \hat{\mu}) = \left[\frac{\partial \hat{x}_T}{\partial \alpha}, \frac{\partial \hat{x}_T}{\partial \sigma}, \frac{\partial \hat{x}_T}{\partial \mu}\right]$$

where the derivatives of the T-year return wave with respect to the parameters are

$$\begin{aligned}
\frac{\partial x_T}{\partial \alpha} &= \frac{\sigma T^{-1/\alpha} \ln T}{\alpha^2 (1 - T^{-1/\alpha}) \ln(1 - T^{-1/\alpha})} \\
\frac{\partial x_T}{\partial \sigma} &= -\ln(-\ln(1 - (1/T)^{1/\alpha}) \\
\frac{\partial x_T}{\partial \mu} &= 1
\end{aligned}$$

and an estimated covariance matrix is given as

$$\widehat{\boldsymbol{\Sigma}} = \left[-\ddot{l}(\hat{\alpha}, \hat{\sigma}, \hat{\mu})\right]^{-1}. \tag{2}$$

4. Fitting of data

In oceanography and ocean engineering, the quantity of *significant wave height* (Hs) is studied. This is defined as the average height of the highest one third wave amplitudes at a given location. Data originate from buoy measurements with Hs reported hourly, calculated as the average of the highest one-third of all of the wave heights during 20-minute sampling periods. Assuming independence between years, the method of block maxima (annual maxima) will be used; the most basic methodology for estimation of return values. It is a well established model in the literature that the limiting extreme-value distribution for data of this type is Gumbel; hence, it is of interest to also study the EG distribution.

4.1. Description of data

Data for a number of buoys are available online from National Data Buoy Center (NDBC). We studied two buoys situated in the North East Pacific: Buoy 46005 (46 N, 131 W) and Buoy 46006 (41 N, 137 W). The time period January 1, 1983, to December 31, 2003, was investigated. For each calendar year, the maximum observation was extracted; hence, for each buoy 21 yearly maxima were found. We assume for simplicity independent samples, for instance no trend present. Moreover, we choose to study calendar years although seasonal years where seasons are kept together might be more advantageous for important estimations. However, the main purpose of this study is to investigate the statistical aptness of the EG distribution: in itself and in comparison to related extreme-value distributions.

Dataset 1 (Buoy 46005): Yearly maxima of Hs (m)

10.70	10.70	7.00	11.30	13.60	11.70	8.20
12.00	9.30	8.80	11.00	11.90	9.20	8.71
9.63	9.87	13.04	9.79	12.26	11.52	12.92

Dataset 2 (Buoy 46006): Yearly maxima of Hs (m)

12.90	8.80	11.80	12.70	11.70	9.10	8.40
9.60	7.20	9.80	10.80	10.10	11.20	9.56
8.25	12.47	16.32	14.65	12.78	14.23	11.21

4.2. Estimation and investigation of fit

In this subsection the EG distribution is fitted to data: parameters are estimated by the method of maximum likelihood, and 100-year return values are computed along with confidence intervals obtained by the delta method.

Optimization was carried out using R, where the results from two methods were examined: a simplex method based on Nelder and Mead (1965) and a quasi Newton method BFGS. It turned out that optimization can be tricky, due to several local minima of the loglikelihood function. In particular, dataset 1 implied problems. These results are presented in Table 1. Note for instance the difference between methods for parameter estimates of Dataset 1 – in particular $\hat{\alpha}$. However, the estimated $\hat{x}_{100}$ will be roughly the same for any of these two parameter settings. For practical purposes, return values are of interest, e.g. for $T = 100$. These quantities and related 95% confidence intervals were computed (by the delta method). For Dataset 1, one finds $\hat{x}_{100} = 14.2$ m (12.4, 16.0), while for Dataset 2, $\hat{x}_{100} = 17.6$ m (13.0, 22.3).

Dataset 1	Nelder-Mead	BFGS
$\hat{\alpha}$	100.31	32.76
$\hat{\sigma}$	7.32	5.74
$\hat{\mu}$	22.49	18.46
Dataset 2	Nelder-Mead	BFGS
$\hat{\alpha}$	2.19	2.20
$\hat{\sigma}$	2.94	2.94
$\hat{\mu}$	11.67	11.68

Table 1. Results from estimation.

Diagnostic plots in the form of QQ plots of residuals after fitting are presented in Figures 1-2. These plots seem to indicate a reasonable fit of the EG distribution; the dots in the QQ plot follow a straight line, etc.

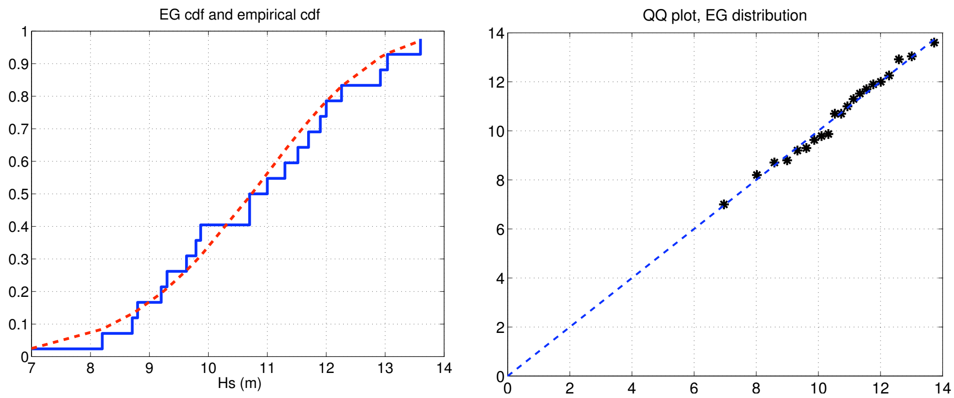

Figure 1: Dataset 1. Left: Empirical and fitted cdf. Right: QQ plot of residuals after fitting.

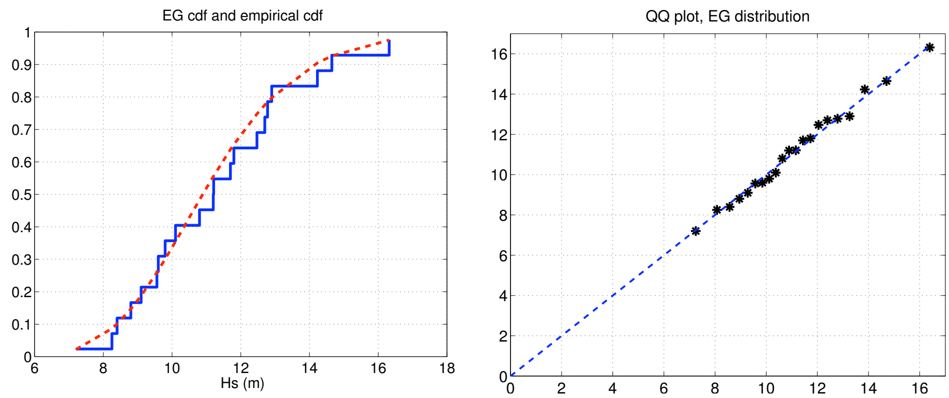

Figure 2: Dataset 2. Left: Empirical and fitted cdf. Right: QQ plot of residuals after fitting.

The sensitivity of data is investigated by removing one observation at a time in the sorted sample and estimating $\hat{\alpha}$, $\hat{\sigma}$ and $\hat{\mu}$. Thus sample i consists of 20 observations with the ith component removed. Denote by $\hat{\theta}_{(i)}$ the ith estimate of a parameter and the sample mean of the estimates by $\hat{\theta}_{(.)}$. Then the jackknife estimate of the standard error is defined by

$$\hat{d}_{\text{jack}} = \sqrt{\frac{n-1}{n}\left(\hat{\theta}_{(i)} - \hat{\theta}_{(.)}\right)^2}$$

and assuming normally distributed estimates, a 95 percent confidence interval for θ can be constructed as $(\hat{\theta} \pm 1.96\,\hat{d}_{\text{jack}})$. The resulting 95% confidence intervals for x_{100} are similar to those computed before: For Dataset 1, the interval (12.6, 15.7) is found; for Dataset 2, (12.9, 22.4).

5. Comparison with other extreme-value distributions

In this section other extreme-value distributions are considered, e.g. the Generalized Extreme Value distribution (GEV)

$$F(x) = \begin{cases} \exp(-(1-\xi(x-\mu)/\sigma)^{1/\xi}), & \xi \neq 0, \\ \exp(-\exp(-(x-\mu)/\sigma)), & \xi = 0, \end{cases}$$

for $x > \sigma/\xi + m$ (when $\xi \leq 0$) and $x < \sigma/\xi + m$ (when $\xi > 0$). As the special case $\xi = 0$ is found the Gumbel distribution. The focus is now on model selection and we investigate the results from two approaches: performance of a likelihood-ratio test and computation of Anderson–Darling distance. In addition, 100-year return values and the related confidence intervals are calculated.

5.1. Comparing models

With competing models as possible explanations of a set of data, a likelihood approach can be used, based on the fact that a log-likelihood (LL) ratio statistic is asymptotically chi-square distributed. The following log-likelihoods were found for the data sets

LL	Dataset 1	Dataset 2
EG	−40.8584	−46.6529
GEV	−40.6890	−46.6315
Gumbel	−42.6171	−46.8145

We note that GEV has the highest log-likelihood in both samples, the EG distribution has the second highest and Gumbel the lowest. These values can be used to test the null hypothesis of Gumbel distribution, since this is a special case of both EG ($\alpha = 1$) and GEV ($\xi = 0$). The statistics 2(LL(EG) − LL(Gumbel)) and 2(LL(GEV) − LL(Gumbel)) are approximately chi-square distributed with one degree of freedom. The following values were obtained from the chi-square distribution:

Test	p value (Data 1)	p value (Data 2)
EG vs Gumbel	0.06	0.57
GEV vs Gumbel	0.05	0.54

For Dataset 1, we note from the table that the hypothesis of Gumbel practically can be rejected at a 5 percent significance level, while for Dataset 2, the hypothesis cannot be rejected. Alternatively, an asymptotic test for shape parameter equal to zero, proposed by Hosking, Wallis and Wood (1985), gives that the hypothesis of Gumbel distribution is rejected for Dataset 1 (p value 0.015).

Next, the Anderson–Darling distance is used to study the differences between the samples and distribution functions. This is given for a distribution function $F_\theta(x)$ by the formula

$$D_{\mathrm{AD}} = \frac{1}{n^2}\sum_{i=1}^{n}(2i-1)\left[\ln F_\theta(x_{(i)}) + \ln(1 - F_\theta(x_{(n+1-i)}))\right] - 1$$

where $x_{(1)}, \ldots, x_{(n)}$ is the ordered sample; see e.g. Boos (1982). The distance D_{AD} is usually considered superior to other distances (like Kolmogorov–Smirnov) with respect to tail behaviours. Values of differences as measured with D_{AD} are given in the table below from which we conclude that for both datasets, EG has the lowest discrepancy. Curiously, the Gumbel distribution in both cases gives a closer fit to data than the GEV.

D_{AD}	Dataset 1	Dataset 2
EG	0.0089	0.030
GEV	1.06	5.75
Gumbel	0.018	0.38

In summary, GEV had the highest log-likelihood in both samples, EG the second highest and the Gumbel distribution the lowest log-likelihood. Considering Anderson–Darling distance D_{AD}, EG gives the closer fit; hence one cannot say whether EG or GEV provides the better fit. The Gumbel distribution had the lowest log-likelihood and second smallest D_{AD}. This indicates that the Gumbel distribution is not as good model as the EG and the GEV. On the other hand, the Gumbel distribution is a special case of GEV and naturally it has a smaller flexibility in modelling the data. In statistics it is not recommended to use more complicated models than needed to describe data adequately and, therefore, this model can be preferable in certain circumstances.

5.2. 100-year return values and confidence intervals

Estimation was carried out for the two data sets by fitting to the three distributions considered. Point estimates and confidence intervals are found in Table 2 from which we conclude that GEV and EG yield point estimates of return values practically the same. However, the confidence intervals for GEV are much wider than those for EG which makes an interesting observation.

Dataset 1		
	x_{100} (m)	Confidence interval
EG	14.2	(12.4, 16.0)
GEV	14.3	(8.2, 20.4)
Gumbel	17.5	(14.6, 20.3)
Dataset 2		
	x_{100} (m)	Confidence interval
EG	17.7	(13.0, 22.3)
GEV	17.6	(2.7, 32.5)
Gumbel	19.0	(15.6, 22.4)

Table 2. Estimation of 100-year return values

5.3. Longer return periods

In this subsection we investigate the behaviour of the three distributions as a function of return period T. In many applications, it is not uncommon to calculate with return periods as high as 10 000 years, corresponding to very small risks.

In Figure 3, the estimates are shown for the two buoys investigated earlier. In the left panel, the results from the Gumbel distribution seem to deviate from the others which would be expected from the likelihood-ratio test earlier. Recall that for Dataset 1, the hypothesis of a

Gumbel distribution is not rejected at the level 0.05. The difference between EG and GEV is about 2 metres for longer return periods, about one metre for return periods less than 1000 years. In the right panel, the differences between the established distributions Gumbel and GEV are quite large for high values of T and the EG distribution gives an intermediate result.

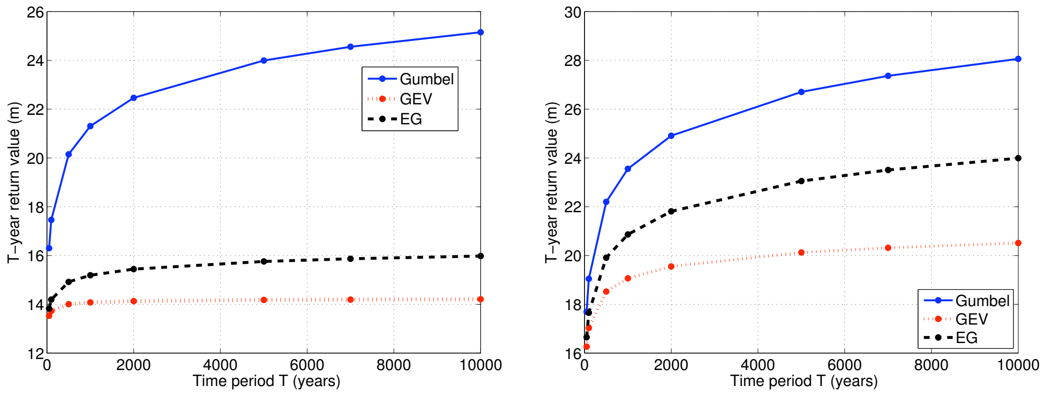

Figure 3: Return values as a function of return period T. Left: Dataset 1; Right: Dataset 2.

However, the authors did also have access to 21 yearly maxima from Buoy 44004, and the corresponding results are shown in Figure 4. Here the estimates by the EG distributions are the highest, although for practical purposes, the results from the Gumbel distribution are close (and $\hat{\alpha} = 0.97$). For this data set, the point estimates are close and the difference in confidence interval may be interesting to study (though not shown here). In summary, based on the analysis of the three datasets, no general conclusion can be drawn about the behaviour of the EG distribution with respect to GEV and Gumbel.

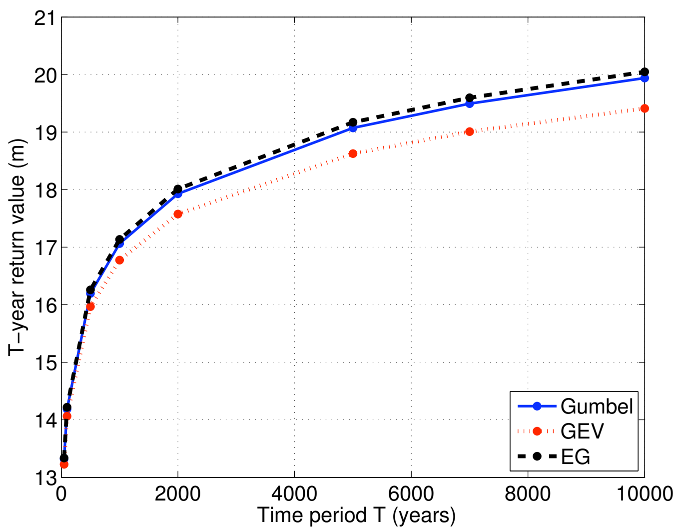

Figure 4: Return value as a function of return period T (Buoy 44004).

6. Concluding remarks

The purpose of this paper was to extend the analysis of the recently introduced EG distribution by investigating estimation of T-year return values for significant wave height. From an applied point of view, such estimations are by no ways trivial and the method of block maxima is the classical, and most simplest, way to proceed. Nevertheless, use of this method is often employed due to its simplicity and may serve as a benchmark when evaluating other, more refined methods (like e.g. threshold methods).

We investigated two datasets with values of about the same order. However, the estimated parameter values possessed a high variability, in particular the estimate of the shape parameter α (100 respectively 2). The authors performed simulation studies and found that it is not unlikely to receive high values of α (for parameter settings used in this paper, α as high as 2000 could be found). However, even with such high values, the estimates of return values are not affected but behave in a stable way. An explanation could be the maximization of the likelihood; the objective function might be flat around the extremum. Moreover, the estimators α, σ, μ are positively correlated. It might be interesting to test other estimation strategies, e.g. the method of moments, possibly in a future work.

Nevertheless, the statistical analysis of datasets indicate that the EG distribution could serve as an alternative to the more well-established GEV distribution (also a three-parameter distribution). In particular, for the data analysed, EG renders narrower confidence intervals than GEV and has for both datasets the smallest Anderson–Darling distance of the distributions examined. The EG distribution deserves further studies, theoretical (estimation methodology) as well as practical (analysis of further datasets).

7. References

Ahuja JC and Nash SW. 1967. The generalized Gompertz-Verhulst family of distributions, *Sankhya, series A 29*, 141-156.

Boos D. 1982. Minimum Anderson-Darling estimation, *Communications in Statistics - Theory and Methods*, 11:24, 2747-2774.

Coles SG, Dixon, MJ. 1999. Likelihood-based inference for extreme value models. *Extremes* **2**: 5-23.

Coles, S. 2001. *An Introduction to Statistical Modelling of Extreme Values*. Springer-Verlag.

Coles S, Pericchi L. 2003. Anticipating catastrophes through extreme value modelling. *Appl. Statist.* **52**: 405-416.

Gupta RC, Gupta PL, Gupta RD. 1998. Modeling failure time data by Lehmann alternatives. *Comm Stat Theory and Methods* **27**: 887-904.

Gupta D, Kundu D. 2001. Exponentiated exponential family: An alternative to Gamma and Weibull distributions, *Biometrical Journal* **43**: 117-130.

Gupta RD, Kundu D. 2007. Generalized exponential distribution: Existing results and some recent developments. *Journal of Statistical Planning and Inference* **137**: 3537-3547

Hosking JRM, Wallis JR, Wood EF. 1985. Estimation of the generalized extreme-value distribution by the method of probability-weighted moments. *Technometrics* **27**: 251-261.

Koutsoyiannis D, Baloutsos, G. 2000. Analysis of long record of annual maxima rainfall in Athens, Greece, and design rainfall inferences. *Natural Hazards* **29**: 29-48.

Kotz S, Nadarajah S. 2000. *Extreme Value Distributions: Theory and Applications*. Imperial College Press: London.

Leadbetter MR, Lindgren G, Rootzén H. 1983. *Extremes and related properties of random sequences and processes*. Springer-Verlag.

Nadarajah S. 2006. The exponentiated Gumbel distribution with climate application, *Environmetrics* **17**: 13-23

Nelder J.A. and Mead R. 1965. A simplex algorithm for function minimization. *Computer Journal* **7**, 308-313.

Rychlik I, Rydén J. 2006. *Probability and Risk Analysis. An Introduction for Engineers*. Springer-Verlag.

Rychlik I, Rydén J, Anderson, CW. 2009. Estimation of return values for significant wave height from satellite data. Preprint - Department of Mathematical Sciences, Chalmers University of Technology and Göteborg University, ISSN 1652-9715; nr 2009:29

Scarf, P. 1992. Estimation for a four parameter generalized extreme value distribution. *Comm. Statist. Theory and Methods* **21**:2185-2201.

Affiliation:

Jesper Rydén
Department of Mathematics
Uppsala University
E-mail: jesper.ryden@math.uu.se