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Wrapped variance gamma distribution with an application to Wind Direction

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Abstract

Since some important variables are axial in weather study such as turbulent wind direction, the study of variance gamma distribution in case of circular data can be an amiable perspective; as such the "Wrapped variance gamma" distribution along with its probability density function has been derived. Some of the other wrapped distributions have also been unfolded through proper specification of the concern parameters. Explicit forms of trigonometric moments, related parameters and some other properties of the same distribution are also obtained. As an example the methods are applied to a data set which consists of the wind directions of a Black Mountain ACT.

Keywords: Directional data,Modified Bessel function, Trigonometric Moments,Watson's U^2 statistic.

1. Introduction

An axis is an undirected line where there is no reason to distinguish one end of the line from the other (Fisher 1993, p. xvii). Examples of such phenomena include a dance direction of bees, movement of sea stars, fracture in a rock exposure, face-cleat in a coal seam, long-axis orientations of feldspar laths in basalt, horizontal axes of outwash pebbles and orientations of rock cores (Fisher 1993). To model axial data, Arnold and SenGupta (2006) discussed a method of construction for axial distributions, which is seen as a wrapping of circular distribution. While in real line, variance gamma distribution was first derived by Madan and Seneta (1990). It has uses in weather statistics, finance, etc. Since some important variables are axial in weather study such as turbulent wind direction, the study of variance gamma distribution in case of circular data can be a better perspective. Although wrapped distribution was first initiated by Levy P.L.(1939), an enormous number of authors (Mardia, 1972), (Jammalmadaka and SenGupta, 2001), (Jammalmadaka and Kozubowski, 2003,2004), (Cohello, 2007),

(Adnan and Roy, 2011, 2012) worked on wrapped distributions. Their works include various wrapped distributions such as wrapped exponential, wrapped gamma, wrapped chi-square, and wrapped weighted exponential distribution etc. However, wrapped variance gamma distribution not yet been sufficiently explored, although this distribution may ever be a better one to model the directional data for weather conditions.
A random variable X (in real line)has variance-gamma distribution if its probability density functions of form

$$f(x) = \frac{\gamma^{2\lambda} e^{\beta(x-\mu)} |x-\mu|^{\lambda-\frac{1}{2}} K_{\lambda-\frac{1}{2}}(\alpha |x-\mu|)}{\sqrt{\pi}\Gamma(\lambda)(2\alpha)^{\lambda-\frac{1}{2}}}; -\infty < x < +\infty \tag{1}$$

where $-\infty < \mu < +\infty$ is location parameter,α,$\lambda > 0$ real parameter, $0 \leq |\mu| < \alpha$, $\gamma = \sqrt{\alpha^2 - \beta^2} > 0$, and$K_\lambda(\bullet)$ is the modified Bessel function of the third kind. However, the general methods of construction of a circular model are: (i) wrapping a linear distribution around a unit circle; (ii) characterizing properties such as maximum entropy, etc.; (iii) one may start with a distribution on the real line and apply a stereographic projection that identifies points X on R with those on the circumference of the circle, say θ.
The circular distribution is a probability distribution whose total probability is concentrated on the unit circle in the plane $\{(cos\theta, sin\theta)\,|0 \leq \theta < 2\pi\}$which satisfies the properties $(i)\, g(\theta) \geq 0$ for all θ, (ii) $\int_0^{2\pi} g(\theta)\, d\theta = 1$, $(iii)\, g(\theta) = g(\theta + 2\pi m)$ where $g(\theta)$ is the probability density function (p.d.f) for the continuous case. Therefore, if X is a r.v. defined on real line, then the corresponding circular (wrapped) r.v. X_w is defined as $X_w = x\,(mod 2\pi)$ and is clearly a many valued function given by

$$X_w(\theta) = \{g(\theta + 2\pi m)\,|m \in Z\}. \tag{2}$$

Thus, given a circular random variable X_w defined in $[0, 2\pi)$, through the transformation $(\theta + 2m\pi)$, with unobservable variable $m \in Z$, we extend the support of X_w to R so that we can apply an in line density function f(x) to the argument$(\theta + 2\pi m)$.

The wrapped circular p.d.f. $g(\theta)$ corresponding to the density function $f(x)$of a linear r.v. X is defined as,

$$g(\theta) = \sum_{m=-\infty}^{\infty} f(\theta + 2\pi m)\,; \theta \in [0, 2\pi). \tag{3}$$

The form of pdf of a wrapped variance gamma distribution is obtained through the concern wrapping in Section 2. Section 3 explicit the characteristics function, trigonometric moments along with related parameters, and alternative pdf of wrapped variance gamma distribution. A method of maximum likelihood given in Section 4 is used in an application in Section 5. As an illustrative example of axial data, we use the wind directions data in a Black mountain ACT. It is recognized that wind directions and its characteristics are important for the maintenance of climate change and wind energy functioning. Finally, the conclusion appears in Section 6.

2. Derivation

Let us consider,

$$\theta \equiv \theta\left(x\right) = x\left(mod2\pi\right). \tag{4}$$

Then θ is wrapped (around the circle) or variance-gamma circular random variables that for $\theta\epsilon[0, 2\pi)$ has probability density function

$$\begin{aligned}
g\left(\theta\right) &= \sum_{m=-\infty}^{\infty} f_X\left(\theta + 2\pi m\right) \qquad (5)\\
&= \frac{\gamma^{2\lambda}}{\sqrt{\pi}\Gamma\left(\lambda\right)\left(2\alpha\right)^{\lambda-\frac{1}{2}}} \sum_{m=-\infty}^{\infty} e^{\beta(\theta+2m\pi-\mu)} \left|\theta + 2m\pi - \mu\right|^{\lambda-\frac{1}{2}} K_{\lambda-\frac{1}{2}}\left(\alpha\left|\theta + 2m\pi - \mu\right|\right)\\
&= \frac{\gamma^{2\lambda}e^{\beta(\theta-\mu)}}{\sqrt{\pi}\Gamma\left(\lambda\right)\left(2\alpha\right)^{\lambda-\frac{1}{2}}} \sum_{m=-\infty}^{\infty} \frac{e^{\beta m2\pi}K_{\lambda-\frac{1}{2}}\left(\alpha\left|\theta + 2m\pi - \mu\right|\right)}{\left|\theta + 2m\pi - \mu\right|^{\lambda-\frac{1}{2}}};\\
&\theta \in \left[0, 2\pi\right), \left(\alpha, \beta\right) > 0, 0 \leq |\beta| < \alpha, \lambda(real)
\end{aligned}$$

where the random variable θ has a wrapped variance gamma distribution $WVG(\mu, \lambda, \alpha, \beta, \gamma)$ with parameter$\mu, \lambda, \alpha, \beta, \gamma$.The figure 1 depicts the probability density function s of the some wrapped variance gamma distributions.

3. Characterization of the density

The trigonometric moment of order p for a wrapped circular distribution corresponds to the value of the characteristics function of the unwrapped r.v. X say $\phi_x(t)$ at the integer value p i.e.

$$\phi(p) = \phi_x(t). \tag{6}$$

In real line the characteristics function of the variance gamma distribution

$$\begin{aligned}
\phi_x(t) &= E(e^{itx})\\
\phi_x(t) &= \left(\frac{\gamma}{\left(\alpha^2 - (\beta + it)^2\right)^{\frac{1}{2}}}\right)^{2\lambda} e^{i\mu_t}.
\end{aligned}$$

Hence, using the equation (6) the characteristic function of the wrapped variance gamma distribution is

$$\begin{aligned}
\phi_x(p) &= \left(\frac{\gamma}{\left(\alpha^2 - (\beta + ip)^2\right)^{\frac{1}{2}}}\right)^{2\lambda} e^{i\mu_p}; p = \pm1, \pm2, ...\\
&= \rho_p e^{i\mu_p} \qquad (7)
\end{aligned}$$

where $\rho_p = \bar{R}_p = \left(\frac{\gamma}{(\alpha^2-(\beta+ip)^2)^{\frac{1}{2}}}\right)^{2\lambda}$ and $\mu_p = \mu p$.

Now, we want to check some properties of the same distribution in the following theorems.

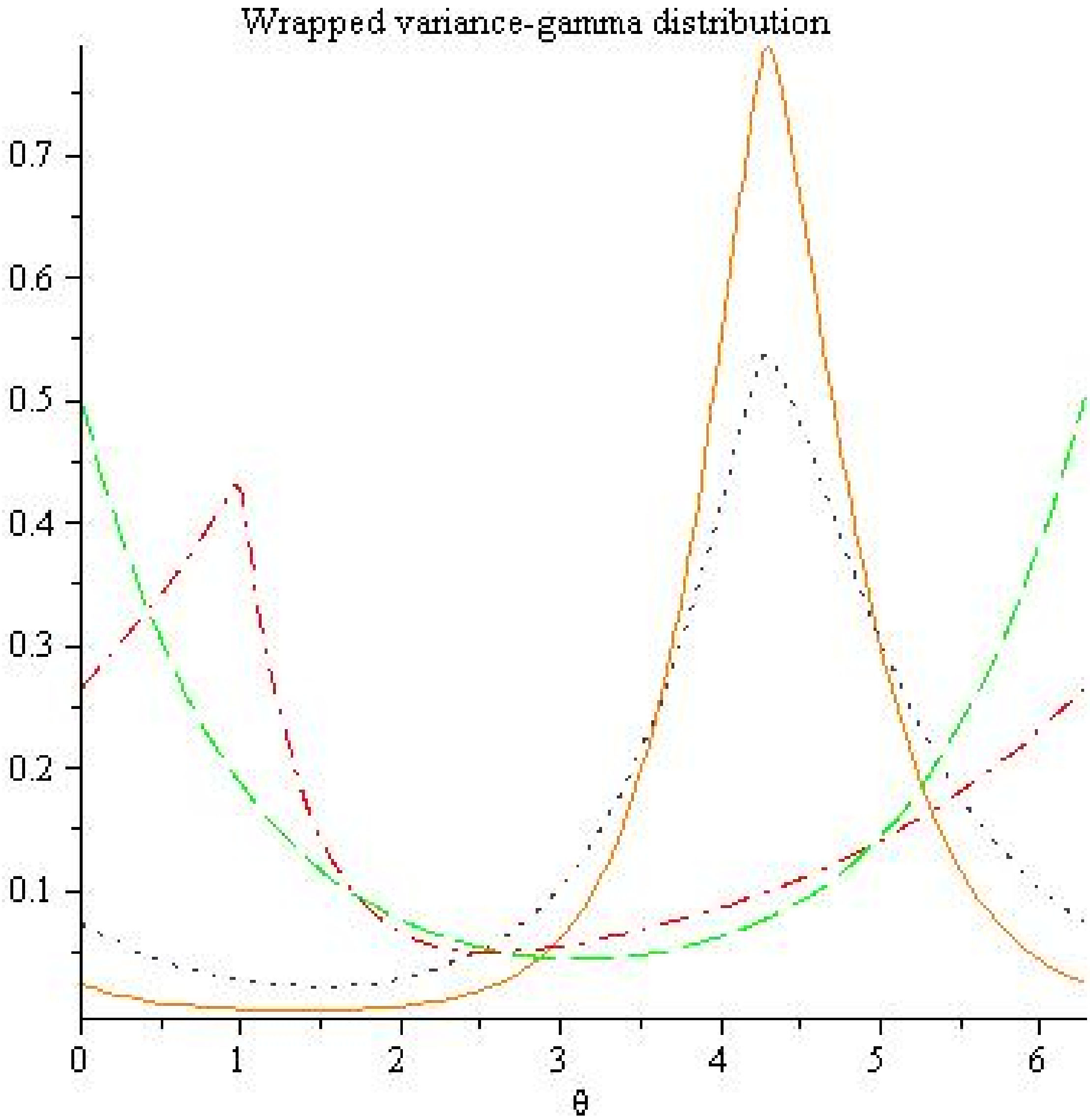

Figure 1: Some Wrapped variance gamma distributions; with *Solidline* : $\mu = -2, \lambda = 1.5, \alpha = 2.5, \beta = 0.2, \gamma = 2.49$; *dotline* : $\mu = -2, \lambda = 1.3, \alpha = 1.5, \beta = 0.2, \gamma = 1.49$; *dashline* : $\mu = 0, \lambda = 1, \alpha = 1, \beta = 0, \gamma = 1$; *dashdotdline* : $\mu = 1, \lambda = 1, \alpha = 1.5, \beta = -1, \gamma = 1.18$.

Theorem 3.1. :If θ_1 and θ_2 are independent angular variables that are wrapped variance gamma distributed with parameters $\theta_1 \sim (\mu_1, \lambda_1, \alpha, \beta, \gamma)$ and $\theta_2 \sim (\mu_2, \lambda_2, \alpha, \beta, \gamma)$ respectively, then $\theta_1 + \theta_2$is also wrapped variance gamma distributed with parameter $\mu_1 + \mu_2$,$\lambda_1 + \lambda_2$,α,β,γ, i.e.$\theta_1 + \theta_2 \sim WVG(\mu_1 + \mu_2, \lambda_1 + \lambda_2, \alpha, \beta, \gamma)$.

Proof: Since θ_1 and θ_2 are follows wrapped variance gamma distribution then the characteristics function(ch.f.) is defined by

$$\phi_{\theta_1}(p) = e^{i\mu_1 p}\left(\frac{\gamma}{(\alpha^2 - (\beta + ip)^2)^{\frac{1}{2}}}\right)^{2\lambda_1}, and$$
$$\phi_{\theta_2}(p) = e^{i\mu_2 p}\left(\frac{\gamma}{(\alpha^2 - (\beta + ip)^2)^{\frac{1}{2}}}\right)^{2\lambda_2}$$

therefore the characteristics function (ch.f.) of $\theta_1 + \theta_2$ is defined by

$$\begin{aligned}
&\phi_{\theta_1+\theta_2}(p) = \phi_{\theta_1}(p)\phi_{\theta_2}(p)\\
&= e^{i\mu_1 p}\left(\frac{\gamma}{(\alpha^2 - (\beta + ip)^2)^{\frac{1}{2}}}\right)^{2\lambda_1} e^{i\mu_2 p}\left(\frac{\gamma}{(\alpha^2 - (\beta + ip)^2)^{\frac{1}{2}}}\right)^{2\lambda_2}\\
&= e^{ip(\mu_1+\mu_1)}\left(\frac{\gamma}{(\alpha^2 - (\beta + ip)^2)^{\frac{1}{2}}}\right)^{2(\lambda_1+\lambda_2)}.
\end{aligned}$$

Since we know that ch.f. satisfies uniquely any distribution function, thus $\theta_1 + \theta_2$ is also wrapped variance gamma distribution with parameters $\lambda_1+\lambda_2, \alpha, \beta, \gamma$i.e. $\theta_1+\theta_2 \sim WVG(\mu_1+\mu_2, \lambda_1 + \lambda_2, \alpha, \beta, \gamma)$.

3.1. Trigonometric moments and related parameters

Using the definition trigonometric moment

$$\phi_p = \alpha_p + i\beta_p;\ p = \pm 1, \pm 2, ...$$

where the non-central trigonometric moments are $\alpha_p = \rho_p\, cos(\mu_p)$ and$\beta_p = \rho_p\, sin(\mu_p)$, So that

$$\alpha_p = \left(\frac{\gamma}{(\alpha^2 - (\beta + ip)^2)^{\frac{1}{2}}}\right)^{2\lambda} cos\,(\mu p)$$
$$\beta_p = \left(\frac{\gamma}{(\alpha^2 - (\beta + ip)^2)^{\frac{1}{2}}}\right)^{2\lambda} sin\,(\mu p)\,.$$

Now according to Jammalamadaka and SenGupta (2001) an alternative expression for the pdf of the wrapped variance gamma distribution can be obtained using the trigonometric moments as

$$g\,(\theta) = \frac{1}{2\pi}\left[1 + 2\sum_{p=1}^{\infty}(\alpha_p\, cosp\theta + \beta_p\, sinp\theta)\right];\theta \in [0, 2\pi)\,.$$

As such substituting the value of $\alpha_p$and $beta_p$ in the immediate above equation we get,

$$= \left[\frac{1}{2\pi}[1+2\sum_{p=1}^{\infty}\left(\left(\frac{\gamma}{(\alpha^2-(\beta+ip)^2)^{\frac{1}{2}}}\right)^{2\lambda} cos\,(\mu p)\,cosp\theta + \left(\frac{\gamma}{(\alpha^2-(\beta+ip)^2)^{\frac{1}{2}}}\right)^{2\lambda} sin\,(\mu p)\,sinp\theta\right)\right]$$

$$= \frac{1}{2\pi}\left[1+2\sum_{p=1}^{\infty}\left(\left(\frac{\gamma}{(\alpha^2-(\beta+ip)^2)^{\frac{1}{2}}}\right)^{2\lambda} (cos\,(\mu p)\,cosp\theta + sin\,(\mu p)\,sinp\theta)\right)\right]$$

$$= \frac{1}{2\pi}\left[1+2\sum_{p=1}^{\infty}\left(\left(\frac{\gamma}{(\alpha^2-(\beta+ip)^2)^{\frac{1}{2}}}\right)^{2\lambda} cosp(\theta-\mu)\right)\right]$$

which is the alternative pdf form of wrapped variance gamma distribution under the trigonometric moments.

Now the central trigonometric moment are $\overline{\alpha_p} = \rho_p cos\,(\mu_p - p\mu_1)$ and $\overline{\beta_p} = \rho_p cos\,(\mu_p - p\mu_1)$. Thus the central trigonometric moments of the same distribution will be

$$\overline{\alpha_p} = \left(\frac{\gamma}{(\alpha^2-(\beta+ip)^2)^{\frac{1}{2}}}\right)^{2\lambda}, and \tag{8}$$

$$\overline{\beta_p} = \left(\frac{\gamma}{(\alpha^2-(\beta+ip)^2)^{\frac{1}{2}}}\right)^{2\lambda} sin(\mu p - p\mu) = 0. \tag{9}$$

Moreover, the resultant length $\rho = \rho_1$, $\rho = \left(\frac{\gamma}{(\alpha^2-(\beta+i)^2)^{\frac{1}{2}}}\right)^{2\lambda}$; Circular mean direction:$\mu = \mu_1$,thus mean direction μ; and Circular variance: $V_0 = 1-\rho = 1-\left(\frac{\gamma}{(\alpha^2-(\beta+i)^2)^{\frac{1}{2}}}\right)^{2\lambda}$.

Circular standard deviation:

$$\sigma_0 = \sqrt{[-2log(1-V_0)]}$$

$$= \sqrt{\left[-2log\left(\left(\frac{\gamma}{(\alpha^2-(\beta+i)^2)^{\frac{1}{2}}}\right)^{2\lambda}\right)\right]},$$

Circular dispersion:

$$\hat{\delta} = \frac{1-\bar{R}_2}{2\bar{R}_2^2} = \frac{1-\left(\frac{\gamma}{(\alpha^2-(\beta+2i)^2)^{\frac{1}{2}}}\right)^{2\lambda}}{2\left(\left(\frac{\gamma}{(\alpha^2-(\beta+i)^2)^{\frac{1}{2}}}\right)^{2\lambda}\right)^2},$$

and Kurtosis:

$$\varsigma_0^2 = \frac{\bar{\alpha}_2^2-\rho^4}{(1-\rho)^2}$$

$$= \frac{\left(\frac{\gamma}{(\alpha^2-(\beta+2i)^2)^{\frac{1}{2}}}\right)^{2\lambda} - \left(\left(\frac{\gamma}{(\alpha^2-(\beta+i)^2)^{\frac{1}{2}}}\right)^{2\lambda}\right)^4}{\left(1-\left(\frac{\gamma}{(\alpha^2-(\beta+i)^2)^{\frac{1}{2}}}\right)^{2\lambda}\right)^2},$$

since from equation (8) for p=2,$\bar{\alpha}_2 = \left(\frac{\gamma}{(\alpha^2-(\beta+2i)^2)^{\frac{1}{2}}}\right)^{2\lambda}$.

The following table (table 1) exhibits various features of the wrapped variance gamma distributions for different chosen values of the parameters $\mu, \lambda, \alpha, \beta, \gamma$ which are also evident from fig 1.

Properties	A	B	C	D
α_p	0.304	0.253	0.500	0.169
β_p	0.728	0.553	0.530	0.349
ρ	0.797	0.609	0.500	0.415
v_0	0.220	0.414	0.500	0.744
σ_0	0.695	1.020	1.174	1.510
ς_0^2	2.487	0.890	0.550	0.379

Table 1: Properties of wrapped variance-gamma distributions with A:($\mu = -2$,$\lambda = 1.5$,$\alpha = 2.5$,$\beta = 0.2$,$\gamma = 2.49$); B:($\mu = -2$,$\lambda = 1.3$,$\alpha = 1.5$,$\beta = 0.2$,$\gamma = 1.49$);C:($\mu = 0$,$\lambda = 1$,$\alpha = 1$,$\beta = 0$,$\gamma = 1$);D:($\mu = 1$,$\lambda = 1$,$\alpha = 1.5$,$\beta = -1$,$\gamma = 1.18$) .

4. Maximum Likelihood Estimation

Let $\theta_1, \theta_2, ..., \theta_n$ be a random sample of size n from a circular distribution with pdf wrapped variance gamma distribution. The likelihood function is

$$l(\Psi) = \prod_{i=1}^{n} \frac{\gamma^{2\lambda} e^{\beta(\theta_i-\mu)}}{\sqrt{\pi}\Gamma(\lambda)(2\alpha)^{\lambda-\frac{1}{2}}} \sum_{m=-\infty}^{\infty} \frac{e^{\beta m 2\pi} K_{\lambda-\frac{1}{2}}(\alpha|\theta_i + 2m\pi - \mu|)}{|\theta_i + 2m\pi - \mu|^{\lambda-\frac{1}{2}}}. \tag{10}$$

Taking log in eqation (10) and log likelihood function will be

$$L(\Psi) = n\left[2\lambda log(\gamma) - \frac{1}{2}log(\sqrt{\pi}) - \frac{1}{2}log(\lambda) - \left(\lambda - \frac{1}{2}\right)log(2\alpha)\right] + \sum_{i=1}^{n}\beta(\theta_i - \mu)$$
$$+log\sum_{i=1}^{n}\sum_{m=-\infty}^{\infty}\frac{e^{\beta m 2\pi} K_{\lambda-\frac{1}{2}}(\alpha|\theta_i + 2m\pi - \mu|)}{|\theta_i + 2m\pi - \mu|^{\lambda-\frac{1}{2}}}$$

$$L(\Psi) = n\left[2\lambda log(\gamma) - \frac{1}{2}log(\sqrt{\pi}) - \frac{1}{2}log(\lambda) - \left(\lambda - \frac{1}{2}\right)log(2\alpha)\right] + \sum_{i=1}^{n}\beta(\theta_i - \mu) +$$
$$\sum_{i=1}^{n}\sum_{m=-\infty}^{\infty} log\left[e^{2\pi m\beta} K_{\lambda-\frac{1}{2}}(\alpha|\theta_i + 2m\pi - \mu|)\right] - \sum_{i=1}^{n}\sum_{m=-\infty}^{\infty}\left(\lambda - \frac{1}{2}\right)log\left[|\theta_i + 2m\pi - \mu|\right]$$

where,$(\mu, \lambda, \alpha, \beta, \gamma)'$ is the parameter vector. Here the maximum likelihood estimates are obtainable by numerical methods.

5. An application to meteorology directional data

The following example shows the effectiveness of wrapped variance gamma distribution (WVG), the data wind directions comprising hourly measurements of three days at a site on Black Mountain, ACT, Australia, was reported in Dr. M.A. Cameron and discussed in Fisher (1993).The data (n= 22, degree) are as given below:
$0, 15, 50, 90, 150, 180, 220, 235, 240, 245, 250, 255, 265, 270, 280, 285, 300, 315, 330, 335, 340, 345.$

The main goal was to provide a regular monitoring of climate change into ACT. Climate change is the greatest threat facing the world today, wind generated electricity is one of a number of ways that we can reduce our reliance on fossil fuel-generated electricity and therefore reduce our greenhouse gas production and limit climate change. The geography and dynamics wind direction this area are important elements of climate system. This study is important given the growing evidence of the ACT climate change and biosphere to global change.

For our illustration, the first step is to find the maximum likelihood estimates of the parameters $\mu, \lambda, \alpha, \beta, and \gamma$ for a WVG distribution. This was carried out with radian measure over $(0, 2\pi)$, and using Maple and R with circstat package, we next find the maximum likelihood estimates of another circular model. This is the generalized vonMises distribution using Matlab. To cheek the goodness of fit of the distributions, WVG, and GvM, we computed the Kuiper, and Watson's U^2 Statistic, for each model. These tests may be found in Jammalamadaka and SenGupta (2001).Table 2 provides the relevant numerical summaries for the two fits with the goodness of fit.

Distributions	MLEs	Kuiper Statistic	Watson's U^2 Statistic	MLL	AIC	BIC	P-value
$WGV(\mu, \lambda, \alpha, \beta, \gamma)$	(4.07,2,0.98,2.1,0.50)	4.35	1.28	-63.40	136.80	133.51	$p < 0.1200$
$GvM(\mu_1, \mu_2, k_1, k_2, \delta)$	(5.02,5.70,1.04,0.0003,0.68)	4.66	1.71	-67.20	144.40	141.11	$p < 0.0010$

Table 2: Summary of fits for the wind derections measured data

From Table 2, it is clear that the generalized vonMises distribution do not provides good models for this data. The smaller value of the four statistic for a WVG distribution indicates a better fit. Thus, the goodness of fit test confirms the superior fit of our proposed model for this data than the generalized vonMises distribution.

6. Conclusion

An expression for the probability density function of wrapped variance-gamma distribution is suggested in the present paper. Some of the other wrapped distributions have also been unfolded through proper specification of the parameters. The alternative form of the pdf of the same distribution is also obtained using trigonometric moments. A class of basic properties is also found here. Modules programmed in matlab, MAPLE-12, R-2.9 with CIRCSTAT package and EXCEL for the concern computations and graphics are available from the authors. We finally show the effectiveness of this model with real data from meteorology.

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