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A MARKOV CHAIN APPROACH ON PATTERN OF RAINFALL DISTRIBUTION

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Abstract

A three-state Markov chain was employed to examine the pattern and distribution of daily rainfall in Uyo metropolis of Nigeria using 15 years (1995-2009) rainfall data obtained from University of Uyo meteorological centre. The Chi-square (χ^2) and WS test statistics were used to test the goodness of fit of Markov chain to the data. Each year was divided into three different periods viz: Pre-monsoon (Jan 1 − March 31), Monsoon (April − Sept. 30) and Post-monsoon (Oct. 1 − Dec. 31). A day was regarded as a dry day if the rainfall was not more than 2.50mm, as a wet day if the rainfall was between 2.51mm to 5.00mm and as a rainy day if rainfall was above 5.00mm. Based on the three conditions of rainfall (dry, wet and rainy) and the statistical techniques applied, it was observed that for the Pre-Monsoon period with Weather Cycle (WC) of 12 days in Uyo metropolis, the expected length (duration) of dry, wet and rainy days were 10 days, 1 day and 1 day respectively. Also, for the Monsoon period with a WC of 5 days, the expected length of dry, wet and rainy days were 2 days, 1 day and 2days while for the Post-Monsoon period with a WC of 8 days, the expected length of dry, wet and rainy days were 6 days, 1 day and 2days respectively.

Keywords: Markov chain, Transition probability matrix, Rainfall Distribution, Weather cycle, Uyo, Nigeria..

1. Introduction

The invaluable demand for rainfall to life has made its study a major concern in many studies Mark (2005); Umoh, Akpan, and Jacob (2013). Meanwhile, examining the variability of rainfall and the pattern of extreme high and low precipitation is very important for the agriculture and the economy of many African countries including Nigeria. It is well established that rainfall is changing on both the global and the regional scale due to global warming Hulme, Osborn, and John (1998). Consequently, information on rainfall probabilities is vital for the design of water supply management, supplementary irrigation schemes and the evaluation of alternative cropping system for effective soil water management plans (Barkotulla, 2009). Such information can also be beneficial in determining the best adapted plant species and the optimum time of seedling to re-establish vegetation on deteriorated rangelands.

Understanding the rainfall distribution is equally necessary for future planning. This is applicable in areas like agriculture, industry, insurance, hydrological studies and the entire planning of the Nigeria economy. The distribution pattern of rainfall rather than the total amount of rainfall within the entire period of time is more important for studying the pattern of rainfall occurrence Garg and Singh (2010). The direct effect of rainfall to the general growth of any nation is another motivation for this study.

Rainfall is the principal phenomenon driving many hydrological extremes such as floods, droughts, landslides, debris and mud-flows; its analysis and modeling are typical problems in applied hydrometeorology Barkotulla (2010). Rainfall exhibits a strong variability in time and space across the globe. Hence, its stochastic modeling is necessary for the prevention of natural disaster.

Fisher (1924) in his study on the influence of rainfall on the yield of wheat in Rothamsted observed that it was the distribution of rainfall during a season rather than its total amount that influences the yield of crops. A number of probability models have also been developed in many studies to describe the pattern of rainfall distributions (Manning 1950; Feyerherm and L.Bark 1967; Kulandaivelu 1984; Phien and Ajirajah 1984; Topalogu 2002). Aneja and Srivastava (1986, 1999) came up with two-state (with two parameters) and three-state (with five independent parameters) Markov chain models to study the pattern of occurrence of rainfall. Purohit, Reddy, Bhaskar, and Chittora (2008) used two-state Markov chain model to find the probabilities of occurrence of dry and wet weeks. Weekly analysis of rainfall at Bangalore was carried out in that work. A follow-up to that work was that of Mangaraj, et.al (2013) who equally used two-state Markov chain to analyse the daily rainfall occurrence.

Garg and Singh (2010) studied the pattern of rainfall at Pantnagar for daily rainfall data of 42 years (1961-2002) using a three-state Markov chain model. They divided each year into three periods and calculated transition probability matrices for seasons of dry, wet and rainy for each of the period.

The present work is motivated as a way of gaining further insight into the pattern of rainfall distribution in the tropics especially the sub-Sahara African region. The study describes the pattern of rainfall distribution in Uyo metropolis in Akwan Ibom State of Nigeria. A Markov chain approach was employed to determine the probability of transitions between the three major daily weather conditions (dry, wet and rainy) in Uyo metropolis. Results from this work would provide a useful guide to policy makers, agriculturists, government and the likes in many areas that might require adequate knowledge of pattern of rainfalls and its distributions in Nigeria.

2. Materials and Methods

Uyo being a city in the South-Eastern part of Nigeria is the capital of Akwa-Ibom State of Nigeria. Akwa-Ibom State is one of the major oil producing states in Nigeria. The city became a capital of Akwa-Ibom State on September 23, 1987 following the creation of Cross-River State. Uyo, with a land mass area of about 115 Square Kilometers and human population of about 250,000 people (NPC, 2006) is geographically located between latitude 7.4^o North and 8.03^o North of the equator and between 4.52^o East and 5.07^o East of Greenwich Meridian James, Akpan, Essien, and Ekpo (2012) respectively. The mean annual rainfall in Uyo metropolis is about 2,480mm with a mean annual temperature of about 27^o C due to its closeness to the coast.

Data on daily rainfall in Uyo metropolis for fifteen years from January1, 1995 to December 31, 2009 were collected from University of Uyo metrological center where all the rainfall data in Uyo metropolis are collated. The daily rainfalls for the three season periods of pre-monsoon, monsoon, post-monsoon were studied and recorded.There were no missing observations in the entire daily rainfall data recorded.

A three-state Markov chain was used to describe the behaviour of rainfall occurrences in this city. The states, as considered were; dry (d), wet (w) and rainy (r). The probability of the process being in a particular state was calculated based on the Markov chain assumption that attaining a state depends on the immediate preceding state only. The conditions of rainfall occurrence for the three states were defined as follows: a day was considered dry if rainfall occurrence on that day was not more than 2.50mm, wet if rainfall occurrence ranges from 2.51mm to less than 5.00mm and rainy if rainfall occurrence ranges from 5.00mm and above. Markov chain, transition probability matrices, probabilities of dry, wet and rainy days in the long run (equilibrium), expected length of season's spell and Weather Cycle (WC) shall be determined in the analysis of daily rainfall data for the periods of Pre-monsoon (Jan 1-March 31), Monsoon (April 1 - September 30) and Post-monsoon (October 1 - December 30) for Uyo metropolis, Nigeria.

A Markov chain is a special category of stochastic process where the state space and index are discrete in nature. It is a discrete-time process for which the future behavior of the process, given the past and the present, only depends on the present and not on the past (Udom 2010). Another property of a Markov chain is that the transition probability matrices are the result of the processes that are stationary in time or space; the transition probability does not change with time or space (Udom 2010).

In the present study, data collected on rainfall occurrences over a sequence of days can be modeled as a three-state Markov chain with state space S = d, w, r. The current day's rainfall was expected to depend only on that of the preceding day; thus, the observed frequency of days of being in a particular atmospheric state j having just left atmospheric state i, i,j = $\{d, w, r\}$ are presented in Table 1 while the associated transition probability matrix is presented in Table 2.

The definitions of the notations used in Table 1 are as provided below;
n_{dd} : Number of dry days preceded by dry days
n_{dr} : Number of rainy days preceded by dry days
n_{wd}: Number of dry days preceded by wet days
n_{rw}: Number of wet days preceded by rainy days
n_{rr} : Number of rainy days preceded by rainy days, and so on.

Table 1: Frequency of days of being in atmospheric state j preceded by atmospheric state i, i, j = d, w, r

		Current Day (j)			
		Dry (d)	Wet (w)	Rainy (r)	Total
Previous Day (i)	Dry (d)	n_{dd}	n_{dw}	n_{dr}	$n_{d.}$
	Wet(w)	n_{wd}	n_{ww}	n_{wr}	$n_{w.}$
	Rainy(r)	n_{rd}	n_{rw}	n_{rr}	$n_{r.}$

More generally,
n_{ij}, i, j = $\{d, w, r\}$ is the number of j days preceded by i days.
Also,
$n_{d.} = n_{dd} + n_{dw} + n_{dr}$: represents the total number of dry days
$n_{w.} = n_{wd} + n_{ww} + n_{wr}$: is the total number of wet days
$n_{r.} = n_{rd} + n_{rw} + n_{rr}$: is the total number of rainy days

The maximum likelihood estimators of P_{ij}, i, j = $\{d, w, r\}$ $\widehat{P_{ij}}$ are given by

$$\widehat{P_{ij}} = \frac{n_{ij}}{\sum_{j=d}^{r} n_{ij}}$$

The transition probability matrix, as given in Table 2 is defined by
$P = P_{ij} = P(j/i)$, where i,j $\in$ S The transition probability matricesP_ij in Table 2 are defined

Table 2: Transition probability matrix of being in atmospheric state j preceded by atmospheric state i, i, j = d, w, r.

		Current Day (j)			
		Dry (d)	Wet (w)	Rainy (r)	Total
Previous Day (i)	Dry (d)	P_{dd}	P_{dw}	P_{dr}	$P_{d.}$
	Wet(w)	P_{wd}	P_{ww}	P_{wr}	$P_{w.}$
	Rainy(r)	P_{rd}	P_{rw}	P_{rr}	$P_{r.}$

as follows;
$P_{dd} = P(d/d)$: Probability of a dry day preceded by a dry day
$P_{dw} = P(w/d)$: Probability of a wet day preceded by a dry day
$P_{wr} = P(r/w)$: Probability of a rainy day preceded by a wet day; and so on, subject to the condition that the sum of probabilities of each row equals to one. That is; $P_{dd}+P_{dw}+P_{dr} = 1$, $P_{wd} + P_{ww} + P_{wr} = 1$ and $P_{rd} + P_{rw} + P_{rr} = 1$

2.1. Goodness-of-Fit Test

To be able to describe the pattern of distribution of rainfall in Uyo metropolis using Markov chain models, it is necessary to establish that the data collected satisfies the basic assumption of the Markov chain that rainfall occurrence on successive days is not independent. The hypothesis to test under this situation is of the form;

H_0: Rainfall occurrences on consecutive days are independent
H_1: Rainfall occurrences on consecutive days are not independent
The appropriate test statistic, as proposed by Wang and Maritz(1990) is the WS statistic given by

$$WS = \frac{S_a + S_b - 1}{\sqrt{V(S_a + S_b - 1)}} \overrightarrow{p} N(0,1) \tag{1}$$

where, $S_a = P_{dd} + P_{ww} + P_{rr}$
$S_b = P_{rd}P_{dr} + P_{wr}P_{rw} + P_{dw}P_{wd} - P_{dd}P_{ww}$
$P_{dd}P_{rr} - P_{ww}P_{rr}$

The test statistic (1) is a goodness-of-fit test that tests the validity of a data set satisfying the current state dependency of the process on the preceding state assumption of Markov chain for a three-state Markov chain as employed here. The variance of $(S_a + S_b - 1)$ in (1) is given by

$$V(S_a + S_b - 1) = (2p_1p_2p_3)[\frac{1}{n_{d.}n_{w.}} + \frac{1}{n_{w.}n_{r.}} + \frac{1}{n_{r.}n_{d.}}]$$

where p_1,p_2 and p_3 are the stationary probabilities calculated as follows: $p_1 = [(1+p) + (1+s)p/q]^{-1}$
$p_2 = [r + ps/q]p_1$
$p_3 = [p/q]p_1$; and
$p = [p_{dr} + \frac{p_{wr}(1-p_{dd})}{p_{wd}}](\frac{1}{1-prr}); q = 1 + [\frac{p_{wr}p_{rd}}{p_{wd}(1-p_rr}]; r = (\frac{p_rw}{1-p_ww}); s = (\frac{p_{rw}}{(1-p_{ww}})$
Based on these test procedures, the null hypothesis that rainfall occurrences on consecutive days are independent would be rejected if $|\widehat{WS}|| \geq Z_{(1-\alpha/2)}$ or if $p(WS > |\widehat{WS}|) < \alpha$ where $Z_{(1-\alpha/2)}$ is the quantile of the standard normal distribution at α level of significance.

2.2. Calculation of Long Run (Equilibrium) Probabilities

Let pi_1, pi_2 and pi_3 be the probabilities of dry, wet and rainy days in the long run (equilibrium). The values of the probabilities pi_i, i=1,2,3 can be determined by the matrix product:

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} \times \begin{pmatrix} P_{dd} & P_{dw} & P_{dr} \\ P_{wd} & P_{ww} & P_{wr} \\ P_{rd} & P_{rw} & Prr \end{pmatrix} \tag{2}$$

This finally gives the estimators of the long run equilibrium probabilities for each of the three periods as:

$$\pi_1 = p_1P_{dd} + p_2P_{wd} + p_3P_{rd}(\text{for dry days}) \tag{3}$$

$$\pi_2 = p_1P_{dw} + p_2P_{ww} + p_3P_{rr}(\text{for wet days}) \tag{4}$$

$$\pi_3 = p_1P_{dr} + p_2P_{wr} + p_3P_{rr}(\text{for rainy days}) \tag{5}$$

Subject to the condition that: $\sum_{i=1}^{3} \pi_i = 1$

2.3. Expected Length of Different Spells of Seasons and Weather Cycle (WC)

A dry Spell: A dry spell of length's 'd' is defined as a sequence of consecutive dry days preceded and followed by wet or rainy days. The probability of 'd', dry days is given by

$$p(d) = (P_{dd})^{(d-1)}(1 - P_{dd}) \tag{6}$$

The expected length of dry spell is given by

$$E(D) = 1/(1 - P_{dd} \tag{7}$$

where $1 - P_{(dd)}$ is the probability of a day being wet or rainy.

A wet spell: A wet spell of length 'w' is defined as a sequence of consecutive wet days preceded and followed by dry or rainy days. The probability of 'w' is given by

$$p(w) = (P_{ww})^(w - 1)(1 - P_{ww}) \tag{8}$$

The expected length of wet spell is given by

$$E(W) = 1/(1 - P_{(ww)} \tag{9}$$

where $1 - P_{(ww)}$ is the probability of a day being dry or rainy. A rainy spell:
A rainy spell of length 'r' is defined as the sequence of consecutive rainy days preceded and followed by dry or wet days. The probability of 'r' is given by

$$p(r) = (P_{rr})^{(r-1)}(1 - P_{rr}) \tag{10}$$

with the expected length of rainy spell obtained as:

$$E(R) = 1/(1 - P_{(rr)}) \tag{11}$$

where $1 - P_{(rr)}$ is the probability of a day being either dry or wet.

Weather Cycle (WC): The weather cycle of each of the periods is given by

$$E(WC) = E(D) + E(W) + E(R) \tag{12}$$

where;
E(WC) is the expected length of the weather cycle; that is, the number of days it will take the process (rainfall) to be found in each of the three seasons (dry, wet and rainy) and return to a particular season after leaving the season,

E(D) is the expected duration of dry days,
E(W) is the expected duration of wet days,
E(R) is the expected duration of rainy days.

3. Analyses and Results

Analyses of rainfall data in Uyo metropolis for 15 years from Jan.1, 1995 to Dec.31, 2009 using a three-state Markov chain model are presented. Hypothesis of dependency of rainfall

Table 3: Estimated Values of WS Test Statistic and the associated p-values

	Pre-Monsoon	Monsoon	Post-Monsoon
WS Statistic value	2.26($p = 0.0119$)	2 ($p < 0.0001$)	15.77 ($p<0.0001$)

occurrences on successive days are tested separately for the three different seasonal periods of Pre-monsoon, Monsoon and Post-moon using hypothesis set stated under section 2.1. Estimated values of WS statistic (1) for the three periods and their associated p-values are presented in Table 3.

Based on the results of goodness-of-fit test in Table 3, it can be concluded that rainfall occurrences on successive days in Uyo metropolis for each of the three seasonal periods of Pre-monsoon, Monsoon and Post-monsoon are dependent ($p < 0.05$), satisfying a major property of the Markov chain model.

Not only this, we present in Tables 3, 4 and 5 the contingency tables of observed and the expected frequencies (in parentheses) of days of being in a particular atmospheric state j having just left atmospheric state i, i, j = {d, w, r} based on the amount of daily rainfall during the Pre-Monsoon, Monsoon and Post-Monsoon periods respectively Uyo Metropolis. By this, possible dependency of the current day atmospheric conditions on the previous day atmospheric conditions is established. In all the three contingency tables, the results of the Pearson Chi-square tests of independence showed significant dependency of current day's weather conditions on the previous day's weather conditions ($p < 0.05$) in Uyo metropolis of Nigeria.

Table 4: Contingency table of Observed (Expected) Frequency of days of being in atmospheric state j having just left atmospheric state i, i, j = {d, w, r} based on the amount of daily rainfall during the Pre-Monsoon period in Uyo Metropolis, Nigeria

		Current Day (j)			Total
		Dry (d)	Wet (w)	Rainy (r)	
Previous Day (i)	Dry (d)	1094 (1077.72)	23 (22.249)	96 (113.023)	1213
	Wet (w)	18 (23.99)	0 (0.4952)	9 (2.516)	27
	Rainy (r)	90 (109.283)	2 (2.256)	22(11.46)	123

Table 5: Contingency table of Observed (Expected) Frequency of days of being in atmospheric state j having just left atmospheric state i, i, j = d, w, r based on the amount of daily rainfall during the Monsoon period in Uyo Metropolis, Nigeria

		Current Day (j)			Total
		Dry (d)	Wet (w)	Rainy (r)	
Previous Day (i)	Dry (d)	887 (854.77)	119 (122.59)	523 (551.64)	1529
	Wet (w)	101 (122.43)	23 (17.56)	95 (79.012)	219
	Rainy (r)	546 (556.80)	78 (79.85)	372 (359.34)	996

Table 6: Contingency table of Observed (Expected) Frequency of days of being in atmospheric state j having just left atmospheric state i, i, j = d, w, r based on the amount of daily rainfall during the Post-Monsoon period in Uyo Metropolis, Nigeria.

		Current Day (j)			Total
		Dry (d)	Wet (w)	Rainy (r)	
Previous Day (i)	Dry (d)	874 (823.45)	40 (50.16)	151 (191.39)	1065
	Wet (w)	38 (47.94)	8 (2.92)	16 (11.14)	62
	Rainy (r)	155 (195.62)	17 (11.29)	81 (45.47)	253

Not only this, the residual analysis of the observed and the expected frequency of days that a particular atmospheric condition was experienced given the amount of daily rainfall in Uyo metropolis was also performed. This is aimed at further confirming the dependency of successive day's weather conditions on the immediate preceding day's situation and this is accomplished via the standardized residuals r_{ij} computed by:

$$r_{ij} = \frac{n_{ij} - E_{ij}}{[E_{ij}(1 - P_{i.})(1 - P_{.j})]^{\frac{1}{2}}} \sim N(0, 1) \tag{13}$$

where n_{ij} is as defined in section 2, $E_{ij} = \frac{n_{i.} \times n_{.j}}{N}$ and $p_{i.} = \frac{n_{i.}}{N}$ is the row ?? marginal probability computed from the contingency Tables 4, 5, and 6. The standardized residuals over the three atmospheric state i, i = d, w, r using statistic (13) are determined for the Pre-Monsoon, Monsoon, and Post-Monsoon periods for Uyo metropolis, Nigeria as presented in Table 7. The plots of these standardized

Table 7: The estimated standardized residuals between the observed and the expected frequency of days that a particular atmospheric condition was experienced given the amount of daily rainfall in Uyo metropolis for the Pre-Monsoon, Monsoon, and Post-Monsoon periods over the possible atmospheric states (i, j), i, j = d, w, r.

State No	State (i,j) on the contingency table	Standardize Residual for Pre-Monsoon r_{ij}	Standardize Residual for Monsoon r_{ij}	Standardize Residual for Post-Monsoon r_{ij}
1	(1,1)	4.4758	1.7980	7.7416
2	(1,2)	0.4836	-0.5081	-3.0757
3	(1,3)	-5.0668	-0.1834	-6.7464
4	(2,1)	-3.6993	-3.0403	-3.0845
5	(2,2)	-0.7172	1.4111	3.1162
6	(2,3)	5.2969	2.3456	1.6451
7	(3,1)	-3.0875	-0.8635	-6.7481
8	(3,2)	-0.1803	-0.2712	1.6679
9	(3,3)	3.4278	1.0494	6.4375
Mean		0.1037	0.1931	0.1060
Standard Deviation		3.6982	1.6525	5.3209

The sinusoidal patterns of the three residual plots in Fig 1 is another clear indication of the dependency of the successive atmospheric states of the process on the immediate preceding states for the three periods of Pre-Monsoon, Monsoon and Post-Monsoon. This further confirm that the basic Markov chain assumption of the dependency of the current state of the process on the preceding state for a three-state Markov chain employed in this study is met.

Based on the observed frequencies in Tables 4, 5, and 6, the transition probabilities for the daily rainfall data in Uyo metropolis are computed as presented in Table 8 for Pre-monsoon, Table 9 for Monsoon and Table 10 for Post-monsoon seasons respectively.

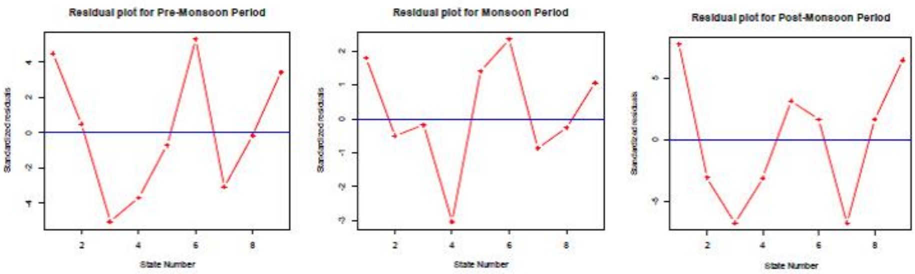

Figure 1: : The plots of the standardized residuals between the observed and the expected frequency of days that a particular atmospheric condition was experienced given the amount of daily rainfall in Uyo metropolis for the Pre-Monsoon, Monsoon, and Post-Monsoon periods. The dependency of successive atmospheric state of the process is clearly evident from systematic patterns of the three residual plots.

Table 8: Estimated transition Probability Matrix of occurrence of rainfall over the three atmospheric conditions in Uyo metropolis for Pre-monsoon season.

		Current Day (j)		
		Dry (d)	Wet (w)	Rainy (r)
Previous Day (i)	Dry (d)	0.90	0.02	0.08
	Wet (w)	0.67	0.00	0.33
	Rainy (r)	0.80	0.02	0.18

$$\begin{pmatrix} 0.90 & 0.02 & 0.08 \\ 0.67 & 0.00 & 0.33 \\ 0.80 & 0.02 & 0.18 \end{pmatrix} \tag{14}$$

Table 9: Estimated transition Probability Matrix of occurrence of rainfall over the three atmospheric conditions in Uyo metropolis for Monsoon season.

		Current Day (j)		
		Dry (d)	Wet (w)	Rainy (r)
Previous Day (i)	Dry (d)	0.58	0.08	0.34
	Wet (w)	0.46	0.11	0.43
	Rainy (r)	0.55	0.08	0.37

$$\begin{pmatrix} 0.58 & 0.08 & 0.34 \\ 0.46 & 0.11 & 0.43 \\ 0.55 & 0.08 & 0.37 \end{pmatrix} \tag{15}$$

Table 10: Estimated transition Probability Matrix of occurrence of rainfall over the three atmospheric conditions in Uyo metropolis for Post-monsoon season.

		Current Day (j)		
		Dry (d)	Wet (w)	Rainy (r)
Previous Day (i)	Dry (d)	0.82	0.04	0.14
	Wet (w)	0.61	0.13	0.26
	Rainy (r)	0.61	0.07	0.32

$$\begin{pmatrix} 0.82 & 0.04 & 0.14 \\ 0.61 & 0.13 & 0.26 \\ 0.61 & 0.07 & 0.32 \end{pmatrix} \tag{16}$$

Finally, the estimated equilibrium state probabilities, the expected length of different spells, the weather cycles and the total numbers of days in each period over 15 years in Uyo metropolis are presented in Table 11. The observed lengths of seasons' spell from the data over the three atmospheric states are equally provided in parentheses in Table 11.

Table 11: Estimated Equilibrium State Probabilities, Expected Length of different Season's Spell, Weather Cycle and Total Number of days of experiencing a given atmospheric condition in Uyo Metropolis of Nigeria. The Observed Lengths of different Season's Spell are given in parentheses

	Current Day (j)			Expected length of Season's Spell				
	Dry (π_1)	Wet (π_2)	Rainy (π_3)	Dry Spell	Wet Spell	Rainy Spell	Weather Cycle	Total No. of Days
Dry (d)	0.89	0.02	0.09	10.00(11)	1.0 0(1)	1.22 (1)	12	1363
Wet(w)	0.56	0.08	0.36	2.38 (2)	1.12 (1)	1.56 (2)	5	2744
Rainy(r)	0.77	0.05	0.18	5.56 (6)	1.15 (1)	1.47 (1)	8	1380

4. Discussion and Conclusions

The patterns of distribution of rainfalls in Uyo metropolis of Nigeria are presented in this study. A three-state Markov chain approach was adopted for this task using the rainfall data collected for fifteen consecutive years. The duration of dry, wet and rainy seasons in the long run (equilibrium), the expected length of spell of each of the three seasons (dry, wet and rainy) and Weather Cycle for each of the periods of pre-monsoon, monsoon and post-monsoon were all determined for Uyo metropolis using the rainfall data collected.

The WS test statistic was used to test the appropriateness of Markov chain techniques on the data based on the assumption that the current state of the process (occurrence of rainfall) depends on the immediate preceding state. From Table 3, the null hypothesis of independence of rainfall occurrence on successive days was rejected ($p < 0.05$); indicating that current day rainfall pattern depends on the preceding day rainfall pattern. This result was supported by the residual plots in Fig 1. This result was equally supported by the closeness of the expected and the observed lengths of seasons' spell as presented in Table 11.

The transition probabilities of rainfall experiences in Uyo metropolis within the three basic seasons of dry, wet and rainy (the state space) for the three periods of pre-monsoon, monsoon and post-monsoon are as reported in the transition probability matrices in Tables 8, 9 and 10 respectively based on the daily rainfall data for 15 years under study.

The three-state Markov chain technique adopted enables easy computation of the equilibrium state probabilities π_1, π_2 and π_3 of a day being dry, wet and rainy respectively as presented in Table 11.

Based on the results of this study, it can be concluded that on the average, for every wet day of each of the three periods, there are about 45 dry days and 5 rainy days for the Pre-monsoon, 7 dry days and 5 rainy days for the Monsoon, 15 dry days and 4 rainy days for the Post-monsoon periods.

Also, on the average, for every rainy day of each of the three periods, there are about 10 dry days and 1 wet day for the Pre-monsoon, 2 dry days and 1 wet day for the Monsoon, 4 dry days and 1 wet day for the Post-monsoon.

From Table 11, it can be observed that the expected lengths of dry, wet and rainy days (rounded up to whole numbers) are 10 days, 1 day and 1 day respectively with a weather cycle of 12 days for the Pre-monsoon; 2 days, 1 day and 2 days respectively with a weather cycle of 5 days for the Monsoon and 6 days, 1 day and 1 day respectively with a weather cycle of 8 days for Post-Monsoon. All these estimates are quite close to the observed seasons' spell over the three weather periods from the data as shown in Table 11. This simply gives more credence to the appropriateness of the Markov-chain model employed in this study.

More generally, findings from this work suggest that agricultural activities (planting) in Uyo metropolis should begin from pre-monsoon period when the weather cycle is relatively longer while the pick of these activities should take place during the monsoon period when the chance of having constant rainfall is highest. Based on the rainfall data analyzed here, the estimated probability of rainy days (π_3) in Uyo metropolis in the long run (equilibrium) during monsoon period is 0.36 which is relatively higher than 0.09 for pre-monsoon and 0.18 for the post-monsoon periods.

Proper applications of the results from this study would help individuals, organizations and government in future planning and policy formulations especially on matters that require adequate understanding of patterns of distribution of rainfalls in Nigeria.

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