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Analysis of Spatial Heterogeneity Using Power Law

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Abstract

We tried to find out the spatial heterogeneity of plant species using power law. A field study was conducted on four grasslands each grazed by a single cow. Grasslands were *Dactylis glomerata* L. dominated grassland without feces (DgF-), *Dactylis glomerata* L. dominated grassland with feces (DgF+), *Veronica arvensis* dominated grassland without feces (VaF-) and *Veronica arvensis* dominated grassland with feces (VaF+). In each grassland, a 50 m line transect was drawn. Each of the four grasslands was surveyed by placing (=100) equal spaced large quadrats (L-quadrats) along the line transect. Each L-quadrats was divided into equal spaced small quadrats (S-quadrats). For each S-quadrat the occurrence of all plant species were recorded. Using the frequency distribution table these data were summarized. The percentage of S-quadrats containing a given species and the variance of each species were estimated. Using the power law the spatial heterogeneity of each species together with community heterogeneity were calculated. We compared degree of heterogeneity index calculated from beta-binomial distribution and from the regression analysis using the power law. The heterogeneity index from the regression analysis was found to be superior to that of the beta-binomial distribution with respect to evaluate the spatial heterogeneity of each species. The per L-quadrat diversity for DgF+ and VaF+ were higher compared to those of the DgF- and VaF-.

Keywords: power law, quadrats, spatial heterogeneity.

1. Introduction

There are many active spatial interactions in ecosystems such as interactions between biotic and abiotic factors, between organisms and environment, and the organisms themselves (Huston 1994). Spatial heterogeneity is formed in ecosystem due to such types of interactions. One of the main factors influencing the spatial heterogeneity in grassland is the excretion of the grazing animal. During grazing, dung, urine and nutrients are unevenly distributed in the grassland (Hakamata 1986), resulting in the spatial heterogeneity and biodiversity.

Measurement of species heterogeneity is the central issue in ecology. Based on the mean and variance of a sample, many indices are used for calculating the spatial heterogeneity. Among them Lloyd's index of mean crowding, Iwao's patchiness regression and Taylor's power law have widely been used (Lloyd 1967; Iwao 1968; Taylor 1961). The power law has paid a particular attention for its mathematical properties (Clauset, Shalizi, and Newman 2009). Sometimes power law leads surprising physical consequences. It has appeared in a diverse range of natural and man-made phenomena. The main aim of this paper is to calculate the spatial heterogeneity of grassland community using the power law.

Madden and Hughes (1995) adjusted Taylor's power law to analyze the number of disease infected plants. They expressed the heterogeneity index of beta-binomial distribution (BBD) in terms of the parameters of the power law (Madden and Hughes 1995). Subsequently the power law has been extended to deal with many plants species from a vegetation survey (Shiyomi, Takahashi, Yoshimura, Yasuda, Tsutsumi, Tsuiki, and Hori 2001). They used power law to determine the frequency of occurrence of plants, the spatial heterogeneity of individual species in a unit area and the community value of spatial heterogeneity in the field (Shiyomi *et al.* 2001). Shiyomi *et al.* (2001) derived the degree of heterogeneity for a specific species in terms of the parameter of power law using regression analysis. Using the degree of heterogeneity for a species they (Shiyomi *et al.* 2001) also calculate an index of heterogeneity for the whole community of a grazing pasture. It is also used to reduce the labour cost and to save the time of any community studies (Shiyomi *et al.* 2001). In this paper, to compare the performance of Madden and Hughes (1995) suggested heterogeneity index with Shiyomi *et al.* (2001) suggested heterogeneity index, the heterogeneity indices were calculated using these two methods.

The structure of this paper is as follows. The general form of binary power law, heterogeneity index of BBD in terms of the parameters of power law proposed by Madden and Hughes (1995) and heterogeneity index proposed by Shiyomi *et al.* (2001) together with the diversity index are briefly presented in Section 2. In Section 3 we investigate the performance of the considered methods using a data set of grassland community. Finally, discussion and conclusion are given in Section 4.

2. Materials and Methods

2.1. Source of data

Data were collected from four grasslands during October 1998 to October 2001 from National Agricultural Research Center in the Tohoku region of Japan (Morioka, Iwate Prefecture). A single cow grazed on each of the four grasslands from May to October each year and fertilizer was not applied. The four grasslands were defined as (i) *Dactylis glomerata* L. dominated grassland without feces (DgF-), in this case the feces was removed everyday, (ii) *Dactylis glomerata* L. dominated grassland with feces (DgF+), in this case the feces was not removed at all, (iii) *Veronica arvensis* dominated grassland without feces (VaF-), from where the feces were removed everyday, and (iv) *Veronica arvensis* dominated grassland with feces (VaF+), from where the feces were not removed at all. Feces were removed once a day with vacuum cleaner. The area of each grassland was about 0.25 ha. In each grassland along a 50 m line transect 100 L-quadrats ($50cm \times 50cm$) as sampling unit were set. Each L-quadrat was divided into small quadrat ($25cm \times 25cm$; S-quadrat). All plant species in each S-quadrat were identified and recorded. Frequencies in all the L-quadrats were calculated for all species. There were 30 to 40 plant species in each of the four grasslands.

2.2. Binary power law

Each L-quadrat contained $n(=4)$ S-quadrats. Frequency distribution table was used in order to estimate percent occurrence of species i per L-quadrat and spatial heterogeneity (unpublished).
The binary form of Taylor's power law was described the linear relation between the observed variance and the binomial variance (theoretical variance) and can be written as (Madden and Hughes 1995):

$$v_{obs} = Av_{bin}^{b} \tag{1}$$

in which A and b are the parameters to be estimated from sample, v_{obs} is the observed sample variance and v_{bin} is the estimated binomial variance.

For each L-quadrat, let $X_i \in \{0, 1, 2, 3, 4\}$ be the number of S-quadrats in which species i was present and X_i follows binomial distribution (Madden and Hughes 1995; Tsutsumi, Shiyomi, and Takahashi 2002). Let p_i be the estimate of the probability of a plant occurred and is given by (Madden and Hughes 1995):

$$p_i = \frac{X_i}{n}$$

The estimate of theoretical variance of p_i is $p_i(1-p_i)/n$.
Suppose that the observed variance of p_i is v_i/n^2 where v_i denotes the variance of observed occurrence counts for species i among L-quadrats.
This law in equation (1) can be written for a whole community with s species as:

$$\frac{v_i}{n^2} = A[p_i(1-p_i)/n]^b \quad \text{for } i \in 1:s \tag{2}$$

By taking logarithms of the both side of equation (2), it can be written as:

$$\log\left(\frac{v_i}{n^2}\right) = \log A + b \log\left[p_i(1-p_i)/n\right] \quad \text{for } i \in 1:s \tag{3}$$

where $\log A$ and b are the intercept and slope of a regression, respectively.
When $A = 1$ and $b = 1$, randomness can be described by the binomial distribution (BD). When $A > 1$ and $b = 1$, there is a aggregated distribution but degree of heterogeneity does not depend on p. When both A and B are greater than 1, the degree of heterogeneity changes with p (Madden and Hughes 1995).

2.3. Power law and Beta-binomial distribution (BBD)

If the plants are aggregatively distributed, the probability of a plant occurred (p_i) is not constant (Madden and Hughes 1995; Tsutsumi *et al.* 2002). Therefore, let p_i be the random variable with the distribution as follows (Madden and Hughes 1995):

$$f(p_i) = \frac{{p_i}^{\alpha_i - 1}(1 - p_i)^{\beta_i - 1}}{B(\alpha_i, \beta_i)} \tag{4}$$

Now, the distribution of X_i is BBD (Madden and Hughes 1995) and the distribution can be written as:

$$\binom{n}{X_i} B(\alpha_i + X_i, \beta_i - X_i + n)/B(\alpha_i, \beta_i) \tag{5}$$

in which α_i and β_i are positive parameters and B is the beta function as:

$$B(\alpha_i, \beta_i) = (\Gamma\alpha_i\Gamma\beta_i)/\Gamma(\alpha_i + \beta_i)$$

For BBD, estimated $\theta_i = 1/(\alpha_i + \beta_i)$ is the index of heterogeneity or aggregation. If $\theta_i = 0$, the distribution is random and the value of θ_i increases with the increasing aggregation. We used t-distribution ($t = \theta_i/s.e.(\theta_i)$) to test the null hypothesis $\theta_i = 0$ (Madden and Hughes 1995). The variance of X_i/n for the BBD is given as:

$$\frac{v_i}{n^2} = np_i(1 - p_i)\frac{(1 + \theta_i n)}{(1 + \theta_i)}$$

Madden and Hughes (1995) describe the relationship between θ_i and the power law parameter as follows:

$$\theta_i = (a - f(p_i)/n)/(f(p_i) - a) \tag{6}$$

with $f(p_i) = [p_i(1 - p_i)]^{1-b}$ and $a = An^{-b}$.

2.4. Power law and Community Heterogeneity

The power law in equation (2) can be expressed by the following linear equation:

$$y_i = a + bx_i + \epsilon_i \quad \text{with } i \in 1 : s \tag{7}$$

where a and b are the constants, $x_i = \log[p_i(1 - p_i)/n]$, $y_i = \log(v_i/n^2)$ and ϵ_i denotes the difference in species i from the regression line i.e. the residual term.
After plotting (x_i, y_i) for species $i \in 1 : s$, the spatial heterogeneity of species i was then determined as follows:

1. for the random pattern of species i the coordinates of species i are on the line $y = x$.
2. for the more heterogeneous pattern than random of species i the coordinates of species i are above the line $y = x$ and
3. for the less heterogeneous pattern than random of species i the coordinates of species i are below the line $y = x$.

The line estimated using regression analysis weighted by p_i, expresses a characteristic of the plant community. We assume that ϵ_i follows a normal distribution with $N(0, \sigma^2)$ under the regression analysis conditions. An estimated regression line located above the line $y = x$ for the whole range of x observed in the survey indicates that the entire landscape tends to be more heterogeneous than that expected from a random distribution. The value of δ_i indicates the degree of heterogeneity or the discrepancy for species i. Now, the δ_i can be defined as:

$$\delta_i = \alpha + (\beta - 1)x_i + \epsilon_i \quad \text{for } i \in 1 : s$$

Using the following equation an index of heterogeneity for whole community can be defined as:

$$\delta_c = \sum_{i=1}^{s} p_i \delta_i / \sum_{i=1}^{s} p_i$$

A large δ_c indicates high spatial heterogeneity at the community level, while a small δ_c indicates that the community forms a low heterogeneity (Shiyomi *et al.* 2001).
Species diversity H', can be calculated by the following equation (Pielou 1977)

$$H' = \sum_{i=1}^{s} p_i \log_{10}(1/p_i)$$

3. Analysis of grassland community data

3.1. Binary power law

The binary power law provided a description of the relationship between the observed and binomial variances of the plants occurring per quadrat. Most of the points were above the binomial line. $R^2 = 0.98$ indicates that 98% of the variation of observed variance can be explained by the variation in the estimated variance (Figure 1). Estimated intercept was $0.59 > 0$ and slope was $1.17 > 1$. Since slope > 1, it is true that for the larger x-value we found a larger difference between the regression line and $y = x$ (Figure 1). It shows that the community exhibits an overall aggregated spatial pattern (Figure 1).
Table 1 shows the estimated power law for the VaF+ grasslands during October 1998 to October 2001. The large value of coefficient of variation R^2 indicates that the deviation of each species from the regression line (community tendency) is small. Based on t-test, we found that estimated slopes and intercepts for all the grasslands were significantly greater than 1 and 0 ($p < 0.05$), respectively. The results describe that most of the points were above the binomial line for all of the regression (Table 1). These results illustrate that the plants in each of the grassland were distributed aggregatively.

3.2. Power law and Beta-binomial distribution (BBD)

Table 2 shows the values of estimated θ for the dominated species of the four grassland. The estimated θ were calculated based on equation 6. The estimated θ gave negative values by this process (Table 2). The significance of these values were tested using t-test (Table 2). These values reveal that each of the species represented low heterogeneity pattern than random pattern (Table 2). The values of estimated θ followed the general trend of estimated θ

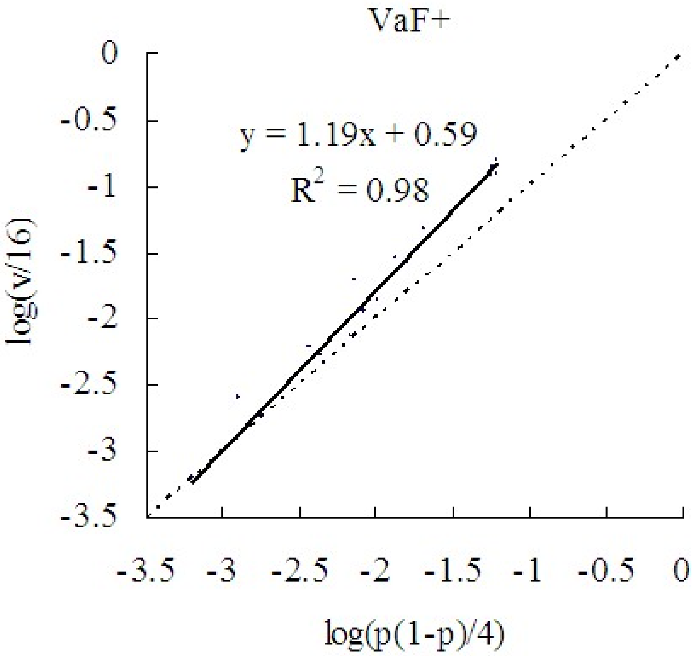

Figure 1: Application of power law to the VaF+ (*Veronica arvensis* dominated grassland with feces) grassland for June 1999. The x and y axis were estimated variance and observed variance, respectively. Solid and dashed lines indicated that the power law estimated from data and the $y = x$, respectively.

Table 1: Results of estimated power law for four grasslands during October 1998 to October 2001

year	Slope		intercept		df	R^2
	ev	se	ev	se		
1998 Oct.	1.21	0.03	0.66	0.06	28	0.99
1999 Jun.	1.19	0.03	0.59	0.08	35	0.98
1999 Oct.	1.18	0.04	0.52	0.10	31	0.96
2000 Jun.	1.17	0.03	0.55	0.07	36	0.98
2000 Oct.	1.17	0.04	0.65	0.09	34	0.97
2001 Jun.	1.17	0.04	0.60	0.10	35	0.95
2001 Oct.	1.19	0.01	0.7	0.1	31	0.97

Ev = Estimated value, se = Standard error, df = Degrees of freedom for the regression, R^2 = Coefficient of determination, Oct. = October, Jun. = June.

with estimated p (Figures 2a, b, c, d). In general, θ increased with p until $p\approx 0.5$ and then decreased at higher p. For all the considered cases we found the estimated θ with a negative value (data were not shown).

Table 2: Estimated parameter θ for the dominated species of four grasslands during October 1998 to October 2001.

Year	Estimated parameters	DgF-	DgF+	VaF-	VaF+
	θ	-1.08	-1.09	-1.09	-1.08
1998 Oct.	$s.e.(\theta)$	0.059	0.039	0.06	0.059
	p-value for $H_0 : \theta{=}0$	$\leq$0.05	$\leq$0.05	$\leq$0.05	$\leq$0.05
	θ	-1.09	-1.08	-1.15	-1.09
1999 Jun.	$s.e.(\theta)$	0.042	0.04	0.05	0.04
	p-value for $H_0 : \theta{=}0$	$\leq$0.05	$\leq$0.05	$\leq$0.05	$\leq$0.05
	θ	-1.09	-1.1	-1.1	-1.01
1999 Oct.	$s.e.(\theta)$	0.04	0.041	0.063	0.04
	p-value for $H_0 : \theta{=}0$	$\leq$0.05	$\leq$0.05	$\leq$0.05	$\leq$0.05
	θ	-1.08	-1.09	-1.13	-1.11
2000 Jun.	$s.e.(\theta)$	0.06	0.037	0.045	0.052
	p-value for $H_0 : \theta{=}0$	$\leq$0.05	$\leq$0.05	$\leq$0.05	$\leq$0.05
	θ	-1.09	-1.10	-1.11	-1.08
2000 Oct.	$s.e.(\theta)$	0.055	0.047	0.063	0.45
	p-value for $H_0 : \theta{=}0$	$\leq$0.05	$\leq$0.05	$\leq$0.05	$\leq$0.05
	θ	-1.08	-1.09	-1.13	-1.08
2001 Jun.	$s.e.(\theta)$	0.056	0.051	0.064	0.04
	p-value for $H_0 : \theta{=}0$	$\leq$0.05	$\leq$0.05	$\leq$0.05	$\leq$0.05
	θ	-1.10	-1.01	-1.10	-1.08
2001 Oct.	$s.e.(\theta)$	0.048	0.043	0.099	0.045
	p-value for $H_0 : \theta{=}0$	$\leq$0.05	$\leq$0.05	$\leq$0.05	$\leq$0.05

DgF- (*Dactylis glomerata* L. dominated grassland without feces), DgF+ (*Dactylis glomerata* L. dominated grassland with feces), VaF- (*Veronica arvensis* dominated grassland without feces) and VaF+ (*Veronica arvensis* dominated grassland with feces). The t-test was used to test $\theta = 0$. Standard error of estimated θ is $(s.e.(\theta))$. Oct. (October), Jun. (June).

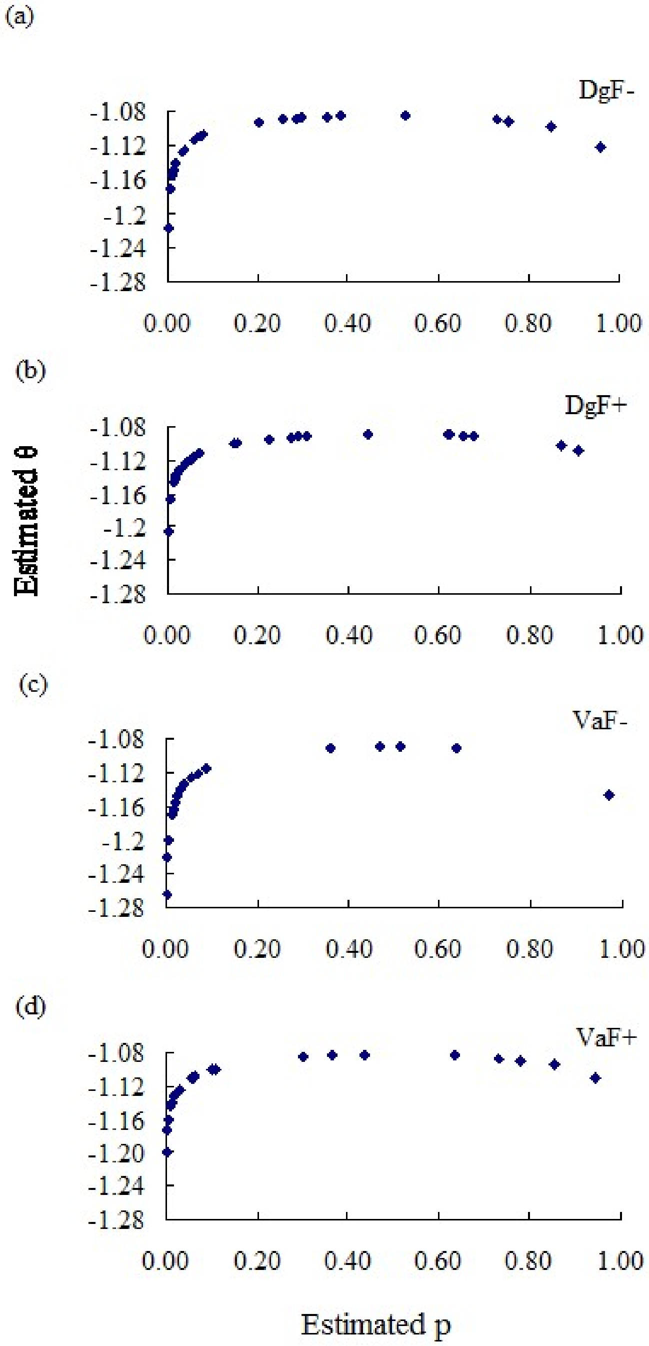

Figure 2: Relationship between estimated θ and estimated p of the four grasslands in October 1998 (a, b, c, d). DgF- (*Dactylis glomerata* L. dominated grassland without feces), DgF+ (*Dactylis glomerata* L. dominated grassland with feces), VaF- (*Veronica arvensis* dominated grassland without feces) and VaF+ (*Veronica arvensis* dominated grassland with feces).

3.3. Community Heterogeneity

Figure 3 illustrates the relationship between the estimated p and the spatial heterogeneity δ, for the 4 geasslands in June, 1999. The DgF- grassland had large δ at relatively high p particularly *Poa pratensis* (Po), *Veronica arvensis* (Va) and *Trifolium repens* (Tr). High δ at low p was found for *Zoysia japonica* (Zy), *Festuca arundinacea* (Fa) and *Plantago lanceolata* (Pl) (Figure 3a).

In the PoF+ grassland *Poa pratensis* (Po), *Trifolium repens* (Tr), *Cerastium glomeratum* (Cg) exhibited high δ at relatively high p. *Zoysia japonica* (Zy), *Anthoxanthum odoratum* (Ao) and *Rumex japonicus* (Rj) revealed high δ at relatively low p (Figure 3b).

In ZyF- grassland, we found that *Veronica arvensis* (Va) had high δ at relatively high p, whereas *Duchesnea chrysantha* (Dc), *Rumex acetosella* (Ra) showed high δ at low p (Figure 3c). In ZyF+ grassland *Zoysia japonica* (Zy), *Veronica arvensis* (Va) had high δ at relatively high p. *Festuca arundinacea* (Fa) and *Agrostis alba* (Aa) showed high δ at low p (Figure 3d). Similar results were also found for the considered time period from October 1998 to October 2001 (unpublished data), and we represent the graph for the October 1998 (Figure 3) only.

3.4. Deviation from the regression line

The regression line in Figure 1 represents the community tendency of spatial heterogeneity in the VaF+ grassland in June 1999. ϵ_i is the deviation between the position of a given species and the regression expresses its divergence from the community tendency in spatial heterogeneity. The relationship between ϵ and p are given in Figure 4.

For $\epsilon \approx 0$ in the DgF- grassland, *Poa pratensis* (Po), *Veronica arvensis* (Va), *Trifolium repens* (Tr), *Erigeron philadelphicus* (Ep), *Paspalum thunbergii* (Pt), *Agrostis alba* (Aa), *Dactylis glomerata* (Dg), *Zoysia japonica* (Zy) showed similar spatial heterogeneity as the community tendency. For $\epsilon > 0$, *Viola grypoceras* (Vg) showed higher spatial heterogeneity at low p (Figure 4a).

In DgF+ grassland *Trifolium repens* (Tr), *Poa pratensis* (Po), *Duchesnea chrysantha* (Dc), *Dactylis glomerata* (Dg), *Erigeron philadelphicus* (Ep), *Viola verecunda* (Vv), *Festuca arundinacea* (Fa), *Zoysia japonica* (Zy), *Anthoxanthum odoratum* (Ao), *Sonchus asper* (Sa), *Erigeron Canadensis* (Ec) had similar spatial tendency as the community tendency, since $\epsilon \approx 0$ (Figure 4b). *Pinus densiflora* (Pd), *Paspalum thunbergii* (Pt) had higher spatial heterogeneity compared to the community tendency for $\epsilon > 0$ at low p (Figure 4b).

In VaF- grassland, at low p *Paspalum thunbergii* (Pt), *Plantago lanceolata* (Pl) had higher spatial heterogeneity than the community tendency (Figure 4c). The spatial heterogeneity of *Veronica arvensis* (Va), *Trifolium repens* (Tr), *Poa pratensis* (Po), *Erigeron philadelphicus* (Ep), *Cerastium glomeratum* (Cg) was similar to the community tendency (Figure 4c).

In VaF+ grassland *Zoysia japonica* (Zy), *Trifolium repens* (Tr), *Veronica arvensis* (Va), *Poa pratensis* (Po), *Rosa multiflora* (Rm) exhibited lower spatial heterogeneity compare to the community tendency because $\epsilon < 0$ at high p (Figure 4d). Where $\epsilon \approx 0$, the spatial heterogeneity of *Cerastium glomeratum* (Cg), *Erigeron philadelphicus* (Ep) was similar to the community tendency. Several species, such as *Agrostis clavata* (Ac), *Lysimachia japonica* (Lj), *Rumex japonicus* (Rj), *Taraxacum officinale* (To) had higher spatial heterogeneity than the community tendency for $\epsilon > 0$ at low p (Figure 4d).

Table 3 shows the values of species diversity index for four grasslands during October 1998 to October 2001. Species diversity of DgF+ and VaF+ grassland was higher than that of DgF-and VaF- grasslands (Table 3).

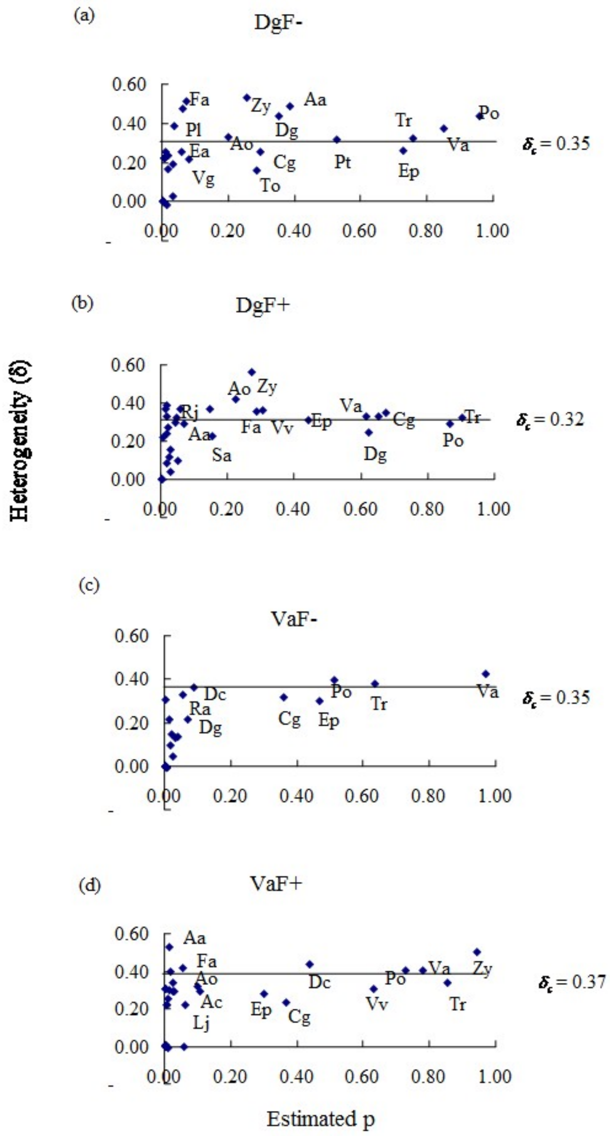

Figure 3: Relationship between spatial heterogeneity (δ) and estimated p for the 4 grasslands in October 1998 (a, b, c, d). DgF- (*Dactylis glomerata* L. dominated grassland without feces), DgF+ (*Dactylis glomerata* L. dominated grassland with feces), VaF- (*Veronica arvensis* dominated grassland without feces) and VaF+ (*Veronica arvensis* dominated grassland with feces). Aa, *Agrostis alba*; Ac, *Agrostis clavata*; Ao, *Anthoxanthum odoratum*; Cg, *Cerastium glomeratum*; Dc, *Duchesnea chrysantha*; Dg, *Dactylis glomerata*; Di, *Digitaria ciliaris*; Ea, *Equisetum arvense*; Ep, *Erigeron philadelphicus*; Fa, *Festuca arundinacea*; Lj, *Lysimachia japonica*; Po, *Poa pratensis*; Pl, *Plantago lanceolata*; Pt, *Paspalum thunbergii*; Ra, *Rumex acetosella*; Rj, *Rumex japonicus* ; Sa, *Sonchus asper*; So, *Sonchus asper*; Sv, *Setaria viridis*; To, *Taraxacum officinale*; Tr, *Trifolium repens*; Va, *Veronica arvensis*; Vg, *Viola grypoceras*; Vv, *Viola verecunda*; Zy, *Zoysia japonica*. δ_c is the community heterogeneity.

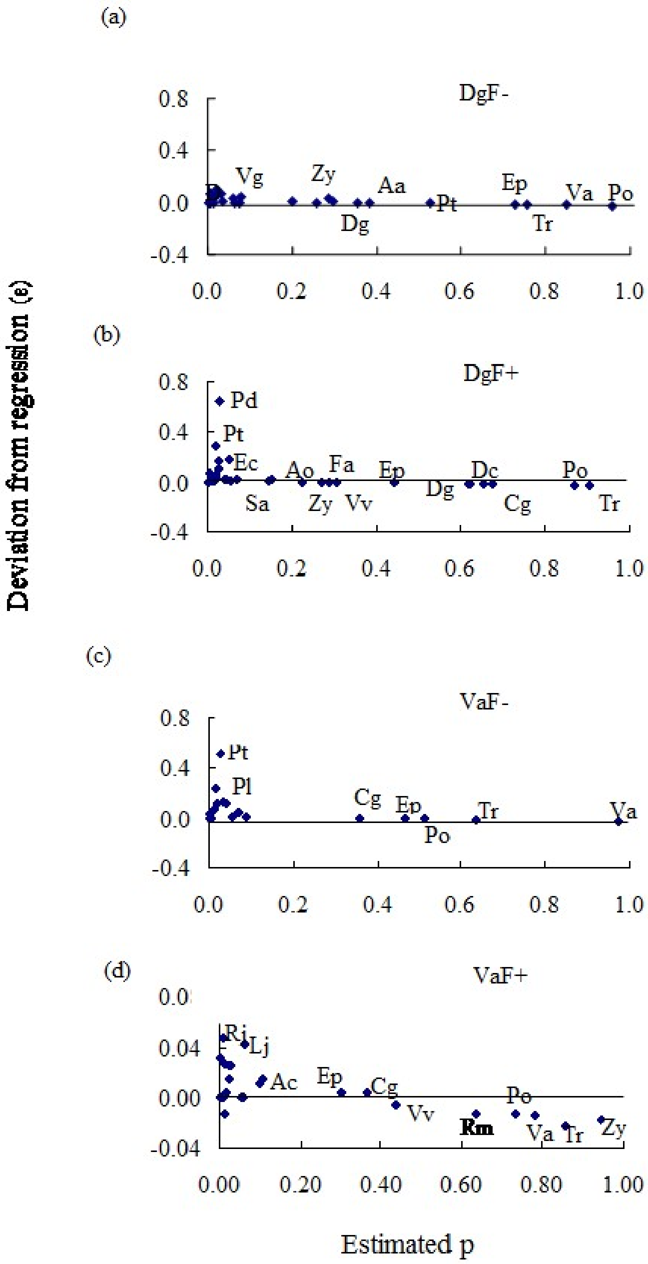

Figure 4: Estimated p ploted against deviation from regression (ϵ) for the 4 grasslands in June 1999 (a, b, c, d). DgF- (*Dactylis glomerata* L. dominated grassland without feces), DgF+ (*Dactylis glomerata* L. dominated grassland with feces), VaF- (*Veronica arvensis* dominated grassland without feces) and VaF+ (*Veronica arvensis* dominated grassland with feces). Aa, *Agrostis alba*;Ac, *Agrostis clavata*; Ao, *Anthoxanthum odoratum*; Cg, *Cerastium glomeratum*; Dc, *Duchesnea chrysantha*; Dg, *Dactylis glomerata*; Di, *Digitaria ciliaris*; Ec, *Erigeron Canadensis*; Ep, *Erigeron philadelphicus*; Fa, *Festuca arundinacea*; Lj, *Lysimachia japonica*; Po, *Poa pratensis*; Pd, *Pinus densiflora*; Pl, *Plantago lanceolata*; Pt, *Paspalum thunbergii*; Ra, *Rumex acetosella*; Rj, *Rumex japonicus*; Rm, *Rosa multiflora*; Sa, *Sonchus asper*; So, *Sonchus asper*; Sv, *Setaria viridis*; To, *Taraxacum officinale*; Tr, *Trifolium repens*; Va, *Veronica arvensis*; Vg, *Viola grypoceras*; Vv, *Viola verecunda*; Zy, *Zoysia japonica.*

Table 3: Species diversity index for four grasslands during October 1998 to October 2001.

Year	DgF-	DgF+	VaF-	VaF+
1998 Oct.	1.13	1.20	0.86	1.02
1999 Jun.	1.12	1.17	0.92	1.06
1999 Oct.	1.13	1.22	0.85	1.00
2000 Jun.	1.12	1.21	0.86	1.04
2000 Oct.	1.13	1.17	0.88	1.04
2001 Jun.	1.10	1.20	0.88	1.05
2001 Oct.	1.12	1.14	0.87	1.03

4. Discussion and conclusion

We used power law as a statistical model to determine the spatial heterogeneity of plants in the four grasslands (Madden and Hughes 1995; Shiyomi *et al.* 2001). The power law provided a good description of the observed variance in the present case (Figure 2), confirming that this law was adequate for analyzing the distribution pattern of plants occurred. The better results of the power law were illustrated that the spatial pattern formed by various species in each grassland was more heterogeneous than would be expected from a random pattern.
Madden and Hughes (1995) studied power law to evaluate the spatial heterogeneity of infected plants. Madden and Hughes (1995) considered that each of N sampling unit contains n plants. The number of infected plants per sampling units was counted. Madden and Hughes (1995) defined p as the probability that a plant is infected. For a random spatial pattern of infected plants between sampling units were defined as np and $np(1-p)$. Madden and Hughes (1995) considered contiguous plants and the disease incidence might be contagious. In that case, the actual mean was np, but the actual variance of the occurrence of plants between cells may be v, not $np(1-p)$. Using v, the δ can be calculated as $\delta = \log(v/n^2) - \log(p(1-p)/n)$ where, n is the number of S-quadrats in an L-quadrat. This δ can also be expressed as $\log(v/(m(1-m/n)))$, where $m = pn$. In this expression v/(m(1-m/n)) is another index of spatial heterogeneity. For the random spatial pattern of occurrences among L-quadrats the value of v/(m(1-m/n)) becomes 1. For more heterogeneous distribution pattern the value of the quantity is > 1. If the quantity is < 1, the distribution pattern is more regular than would be expected with a random distribution (Shiyomi and Yoshimura 2000). Therefore, δ is a reliable index of measuring spatial heterogeneity.
In this study, Madden and Hughes (1995) suggested heterogeneity index θ and Shiyomi *et al.* (2001) suggested heterogeneity index δ were used to compare the degree of heterogeneity (Table 2 and Figure 3). The values of θ vary from $(-1/n)$ to ∞ (Madden and Hughes 1995). Although the estimated θ based on equation (6) followed the general trend of estimated θ with p (Table 2 and Figure 2), all the values of estimated θ were significant. The estimated θ showed that distridution pattern of the plants occurred is low heterogeneous in these four represented grasslands than would be expected with a random distribution.
On the contrary, the δ showed the plants in the four represented grasslands were heterogeneously distributed. With spatial heterogeneity, it was possible to calculate the ϵ. Again using the power law it is possible to compare the heterogeneity of species within the grassland by δ_i and analysis the heterogeneity of the whole community by δ_c. We can determine the

characteristics of each population as well as the whole community, using the parameters p, δ, ϵ and H' obtained from the survey.

The results could be summarized as Shiyomi and Yoshimura (2000) suggested heterogeneity index δ can be applied widely to study the plant community with short height such as grassland community. Although few studies have applied this method, use of the power law with the parameter values of p and δ is a time and labor saving tool for understanding the heterogeneity of grassland community.

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