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Frailty Models for the Control Time of Wildland Fires in the Former Intensive Fire Management Zone of Ontario, Canada

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Abstract

Using the control time of a forest or wildland fire, defined as the time from the start of suppression action to the time that it is declared under control, we extend the analysis from Morin *et al.* (2015) to investigate spatial trends in forest fire survival probability across Ontario's Intensive Fire Management Zone for the period 1989 to 2004. The fire management compartments (FMCs) described in Woolford *et al.* (2009) form the spatial units of analysis. Spatial differences are explored in our study region by using proportional hazards shared frailty models which incorporate a random effect to modify the hazard for fires within each FMC. Estimates of this excess risk are used to visualize spatial patterns. We show that the frailty models achieve better fit, as compared to the models without frailty terms, and that the model assumptions are suitable for these data. Visualizing the estimated FMC-specific frailties suggest the following: lightning-caused fires in a region of northwestern Ontario have experienced shorter control times than comparable lightning fires that occur elsewhere; and, people-caused fires in that same region in northwestern Ontario as well as a region of southern Ontario may also have experienced shorter control times than comparable people-caused fires that have occurred elsewhere.

Keywords: Cox proportional hazards, forest fires, random effects, shared frailty, survival, time-to-event modelling, wildfires.

1. Introduction

Forest and wildland fires have had a significant detrimental impact on people, property and forest resources in recent years and those impacts are not expected to subside anytime soon (Moritz *et al.* 2014; Stocks and Martell 2016). Although fire is a natural component of many forest ecosystems, forest fire management agencies strive to reduce the area burned by destructive fires. These fires pass through several stages of control that begin when they are first reported and classed as being "Being Investigated" (BIV) or "Not Under Control" (NUC), then "Being Held" (BHE), "Under Control" (UCO) to finally being declared "OUT". A BHE fire is one for which "with currently committed resources, sufficient suppression action has been taken that the fire is not likely to spread beyond existent or predetermined boundaries under prevailing and forecasted conditions" (CIFFC 2003). The longer they remain in the BIV, NUC or BHE states, the longer they remain exposed to fire weather conditions that may be conducive to further growth and destruction. Once a fire is classes as UCO, it is viewed as "having received sufficient suppression action to ensure no further spread of the fire" (CIFFC 2003).

The need for fire suppression resources varies both temporally and spatially. Strategic planning requires fire managers to decide where to home base their suppression resources at the beginning of each fire season as well as daily deployment decision-making associated with deciding how to redeploy them on a daily or hourly basis. Knowledge concerning areas where more values at risk coincide with long times required to bring fires under control can and should inform strategic planning decision-making. Wildland fire management agencies can use such information to selectively allocate suppression resources to the fires which are more likely to survive longer in an uncontrolled state. Under stressed systems, when prioritizing fires is most crucial, a modelling technique for the control time of fires could be an invaluable asset to the strategic, tactical and operational planning systems.

Morin *et al.* (2015) presented an overview of survival analysis methods in the context of an analysis of historical records of forest fires that occurred in the former Intensive Fire Management Zone in the Province of Ontario, Canada during the period 1989 through 2004. Cox proportional hazards (PH) models with fire weather and management covariates were employed to quantify the hazard rate of a suppressed fire being brought under control, with separate models fit to lightning and people-caused fires. Regardless of the cause of ignition, the size at initial attack and measures of short-to-moderate fuel moisture were significant predictors for both causes of ignition. Response time was found to be a significant predictor for lightning-caused fires. The time of day at the onset of initial attack efforts along with the Drought Code, a long-term fuel moisture code (see e.g., Wotton 2009), were found to be significant predictors for fires started by people. Our work extends that of Morin *et al.* (2015). We incorporate frailties, namely a set of site-specific random effects, that are used to account for unobserved or unmeasured factors that may also be affecting fire control times in a given area. Visualization of the frailty terms allows us to explore for spatial trends in the control times of wildland fires.

We note that the analysis of time-to-event or survival data is common in the biostatistical literature (e.g., Fleming and Lin 2000), in which frailty models are often employed to capture the spatial correlation between observations (e.g., Banerjee *et al.* 2003). Frailty models are also sometimes used in ecological studies; examples include the probability of tree establishment from germinated seedlings (Goheen *et al.* 2010) and the probability of bird nest survival

(Liebezeit *et al.* 2009). Posterior estimates of random effects are commonly visualized to explore for spatial patterns. For example, geographic distributions of disease risk have been studied by extracting the posterior estimates of spatial random effects. In such studies, hotspots of high risk geographic areas are identified and targeted for resources with the goal of minimizing the "noise" in the map of the spatial random effects (e.g., Lawson 2001). In a similar fashion, we use spatially-referenced frailty terms (random effects) to investigate for spatial patterns in the control time of wildland fires, after accounting for key factors that influence the "lifetimes" of such fires over a set of polygons that partition our study area.

The remainder of our paper is structured as follows: we begin the next section by describing our data and study region and presenting some exploratory analyses. We then describe the framework for modelling the control time of fires using frailty models in Section 3. Cox PH shared frailty models are fit and spatial patterns are examined using choropleth maps in Section 4. The goodness of fit of these models, along with an examination of the modelling assumptions are also presented. We conclude the study with a discussion of our results, the associated forest fire management implications and future work in Section 5.

2. Data and Study Region

Our study region consists of the former Intensive Fire Management zone of Ontario. This region covers diverse forest landscapes which can result in differences in characteristics of fire regimes, such as spatial differences in seasonal patterns in fire occurrence risk baselines as documented by Woolford *et al.* (2009). In this paper, we extend the Cox PH models presented in Morin *et al.* (2015) to account for spatial heterogeneity by assigning a common random effect term to observations within small polygons that form a spatial partition of the region using shared frailty models. The spatial units of analysis are a set of *fire management compartments* (FMCs). As described in Woolford *et al.* (2009), researchers in the Fire Management Systems Laboratory in the Faculty of Forestry at the University of Toronto created this partition by overlaying the map of Fire Management Zones in Ontario (see OMNR 2004) with a digital map of ecoregions (see Ecological Stratification Working Group (Canada) 1996) and then further subdividing some of the resulting polygons to create a final set of FMC polygons that were approximately the same size. Each FMC can be assumed to be approximately internally homogeneous with respect to ecological characteristics such as fuel, weather and topography, as well as fire management strategy. The study area along with the set of FMC polygons are shown in Figure 1.

Historically, the strategy in this area was to actively suppress all fires that were reported as soon as resources were available. However, changes to the number of zones and each zone's management strategy occurred in 2004 (OMNR 2004). Consequently, we focussed our analysis on the period prior to this change to both avoid management strategy changes as possible confounders and so that we can compare our results to the aspatial models of Morin *et al.* (2015).

Our fire data consists of the 7,303 lightning and 8,311 people-caused forest fires included in the historical fire management records provided by the Ontario Ministry of Natural Resources and Forestry that occurred in the Intensive Fire Management zone of Ontario during the period from 1989 through to 2004.

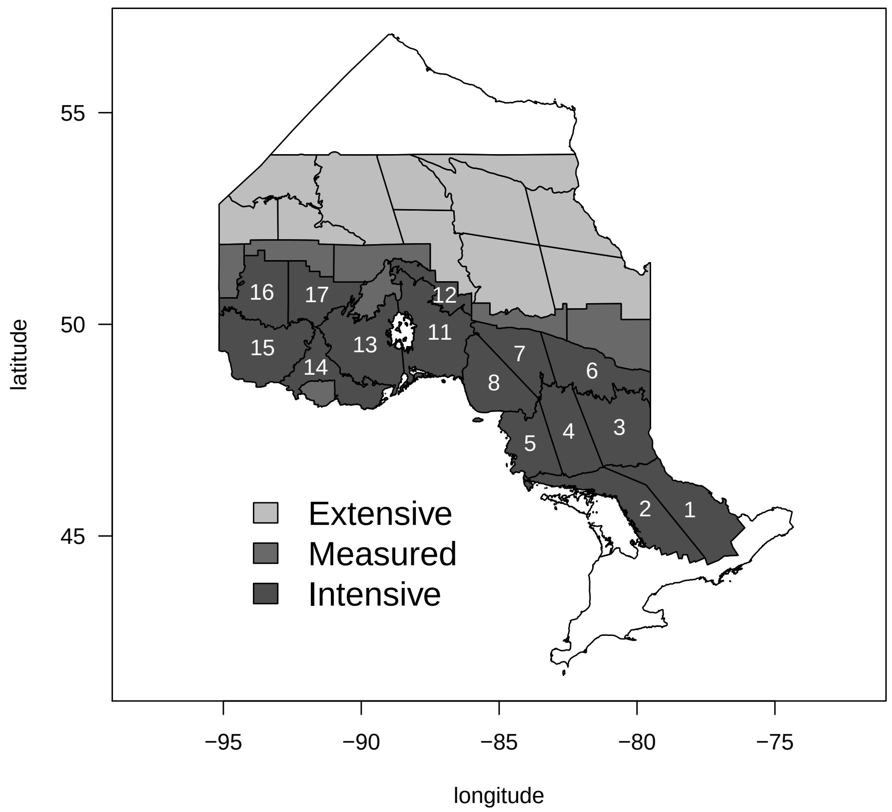

Figure 1: Ontario's Forest Fire Management Zones (shaded areas) prior to 2004 along with the fire management compartment (FMC) partition (polygons) as described in Woolford *et al.* (2009). The set of FMCs in the Intensive Fire Management Zone that comprise of our study area are numbered.

We define the control time of a fire as the time from the onset of initial attack action to the time that the fire is declared under control. During this time interval, suppression crews are actively suppressing the fire to quickly and safely bring it to a state of "Being Held" and then "Under Control" at a reasonable cost. Figure 2 displays the number of lightning and people-caused fires which occurred in each of the compartments during our study period, where the width of the bars represent the median control times of these fires. Note in particular, that FMC 15 experienced the most lightning-caused fires. Note also that those fires have the shortest median control time; we return to discuss this later in this paper. Figure 3 displays the smoothed Kaplan-Meier (KM) (e.g., see Lawless 2003) estimates of the survival probabilities of lightning and people-caused fires by compartment. While all the survival curves in this figure have similar shapes, they clearly exhibit region-specific differences; these differences seem to be more pronounced for lightning-caused fires, as compared to people-caused fires.

To account for some of this region-specific heterogeneity, fire weather components of the Canadian Forest Fire Danger Rating System (CFFDRS) (Stocks *et al.* 1989), namely the Fine Fuel Moisture Code (FFMC), the Initial Spread Index (ISI), and the Drought Code (DC), were included in the frailty models as fixed effects. The FFMC is representative of the moisture content of the dead fine fuels on the surface of the forest floor, while the ISI is a measure of the potential rate of spread of a fire. The DC is representative of the moisture in the deep organic layers of the forest floor. Additional fixed effects included in the frailty models are the *response time*, defined as the time interval (hours) from when a fire is reported to the start of initial fire suppression action, as well as the size (ha) of the fire at the start of initial attack; longer response times are associated with larger fires at initial attack, which are hypothesized to result in longer control times as they are more difficult to suppress. Stratification by the time of day that initial attack begins, categorized as morning, afternoon, or evening as defined in Morin *et al.* (2015) and similar factor variables (with morning as the baseline) are used in the models for lightning and people-caused fires, respectively. This was done to ensure that the plateaux features visible in the left-tails of the KM curves, caused by suppression being suspended overnight, are appropriately modelled (see Morin *et al.* 2015, Figure 4).

3. A Framework for Modelling the Control Time of Fires

Let T be a random variable representing the control time of a fire. Its observed value, t, is the time between the onset of initial attack action and the time that the fire is declared under control. In the following framework, the hazard, $\lambda(t)$, represents the instantaneous rate at which a fire will be declared under control at time t, conditional on it having survived up to that point in time (Lawless 2003).

3.1. The PH Shared Frailty Model

The purpose of frailty models is to describe the excess risk, commonly referred to as frailty, for distinct categories (Therneau and Grambsch 2000). The main ideas in our modelling are that: not all fires are assumed to have the same underlying survival time distribution; there could be other unobserved or unmeasurable factors impacting that distribution; and an unmeasured random effect is used to account for this source of heterogeneity between survival times of fires in different geographic regions (Hosmer *et al.* 2008).

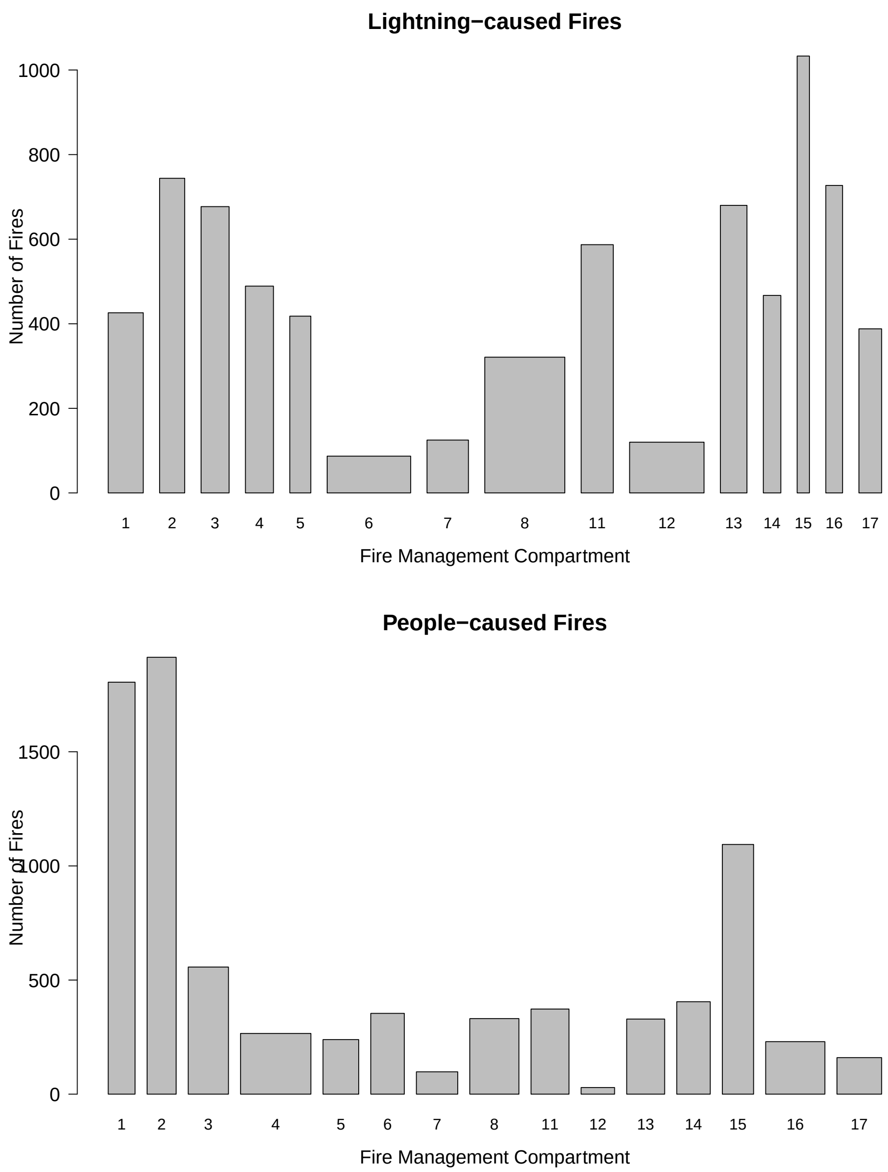

Figure 2: The frequency (height of bars) and median control time (width of bars) of lightning (top panel) and people-caused (bottom panel) fires by fire management compartment.

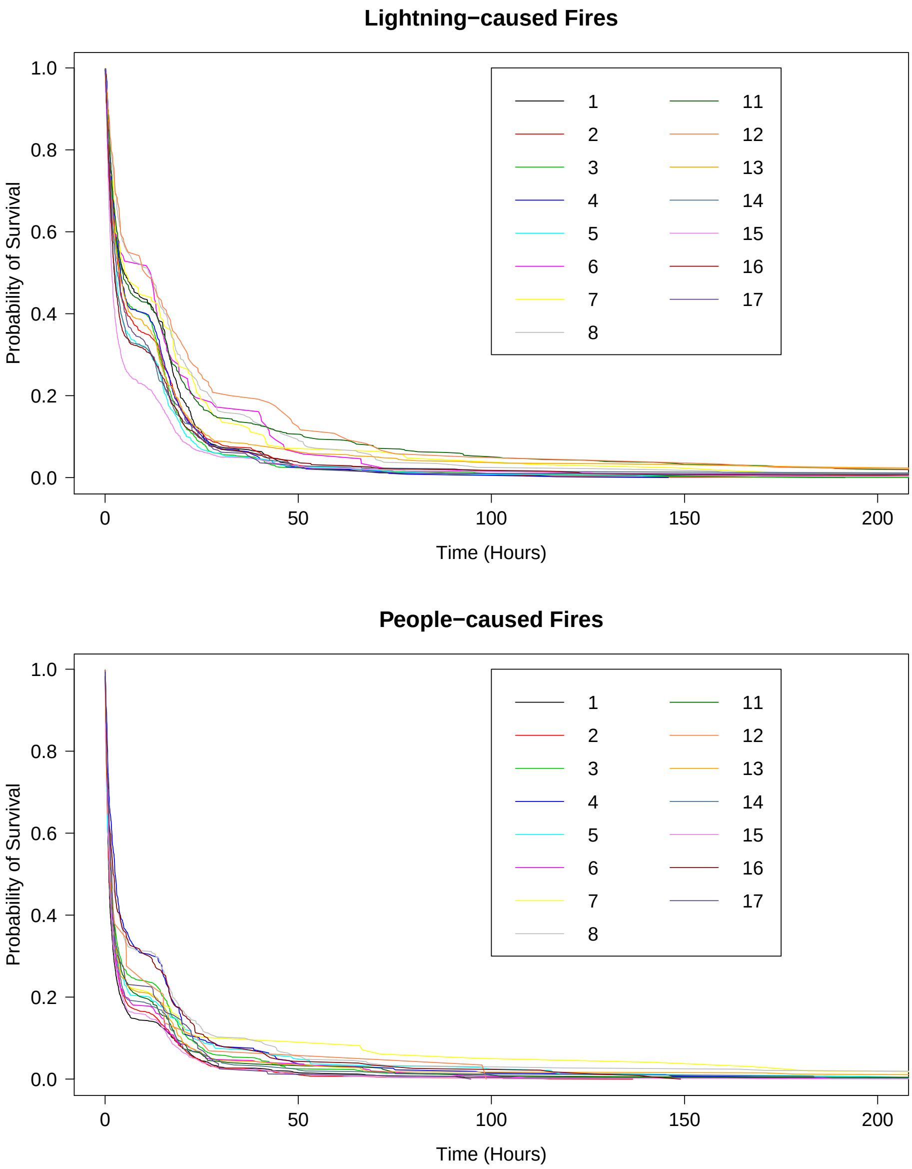

Figure 3: Smoothed Kaplan-Meier estimates of the probability of survival of lightning (top panel) and people-caused (bottom panel) fires by fire management compartment (coloured lines) in the former Intensive Fire Management zone of Ontario.

In the shared frailty model each fire belongs to only one of the distinct FMCs and all fires within an FMC share a common random effect term; these random effects are assumed to be independent of each other (Therneau *et al.* 2003). Fires in the same FMC share the same random effect distribution to account for spatial dependency among fires within the same geographic region. Conditional on the FMC compartments, we assume that the control times of the fires independent since each FMC can be viewed as being approximately internally homogeneous with respect to ecological characteristics such as fuel, weather and topography, as well as fire management strategy. The FMC-specific random effects are referred to as the frailty terms. These frailty terms render each individual fire's hazard bigger or smaller than the baseline hazard (Lawless 2003). The hazard rate from the PH shared frailty model takes the form

$$\lambda_{ij}(t) = \lambda_g(t) e^{\mathbf{x}'_{ij}\boldsymbol{\beta}+\omega_i}$$

where i indexes the FMCs, j indexes the fires in FMC i, g indexes any stratification (e.g., we stratify by time-of-day in our model for the control time of person-caused fires as was also done in Morin *et al.* (2015)), λ_g is the baseline hazard rate of fires in stratum g, $\boldsymbol{x}'_{ij}$ is the transpose of the covariate vector with associated parameter vector $\boldsymbol{\beta}$ and ω_i is the random effect term of the fires in FMC i. Note that if ω_i is set to 0 for all FMCs (i.e., there is no spatial effect), then this hazard rate is equivalent to the aspatial Cox PH model used in Morin *et al.* (2015).

When fitting such models, the random effects are included in the hazard rate as unobserved continuous covariates which are assumed to follow a probability distribution with a fixed mean and unspecified variance. The choice of the distribution of the random effect is based on the structure present in the data being modeled. In practice, the most common choices include the gamma distribution with mean 1 and the normal distribution with mean 0. The latter is used in this paper as it allows flexibility and we later demonstrate that this normal frailty term is appropriate for our analysis. The parameters in the Cox PH frailty model are estimated using a two-step iterative procedure to maximize the penalized partial likelihood, the details of which may be found in Ripatti and Palmgren (2000).

4. Results

The results presented in this section were produced using the **coxme** package of Therneau (2012) in `R` (R Core Team 2014). We first present the results of fitting shared frailty models to lightning-caused fires and then those for people-caused fires, followed by model assessment.

4.1. Exploring for Spatial Patterns Using Shared Frailty Models

Semi-parametric Cox PH shared frailty models with zero-mean, normally distributed random effects are fit to lightning and people-caused fires with resulting random effect variance estimates of 0.0295 and 0.0189, respectively. The parameter estimates, standard errors and p-values of the fixed effects from these fitted models are displayed in Table 1. For comparatison, the fixed effects used in these models and the time-of-day stratification for people-caused fires are the same as in the Cox PH models reported in Morin *et al.* (2015). The negative parameter estimates indicate a decreased hazard rate with increasing covariate values; equivalently this is indicative of the probability of survival increasing with the covariate in question.

Table 1: Parameter estimates, standard errors and p-values of the fixed effects from the fitted stratified proportional hazards shared frailty model of lightning-caused fires (top panel) and proportional hazards shared frailty model of people-caused fires (bottom panel).

Lightning-caused Fires		
Parameter	**Estimate (Std. Error)**	**P-Value**
Response time	-0.0091 (9.2×10^{-4})	< 0.001
FFMC	-0.0090 (1.1×10^{-3})	< 0.001
Size at initial attack	-0.0002 (8.8×10^{-5})	0.0190
ISI	-0.0149 (5.1×10^{-3})	0.0034

People-Caused Fires		
Parameter	**Estimate (Std. Error)**	**P-Value**
Size at initial attack	-0.0252 (2.4×10^{-3})	< 0.001
ISI	-0.0286 (3.9×10^{-3})	< 0.001
DC	-0.0010 (9.4×10^{-5})	< 0.001
FFMC	-0.0061 (1.3×10^{-3})	< 0.001
$I_{\text{afternoon initial attack}}$	-0.1316 (3.5×10^{-2})	< 0.001
$I_{\text{evening initial attack}}$	-0.1574 (3.8×10^{-2})	< 0.001

The posterior estimates of the random effects, $\hat{\omega}_i$, are displayed in the choropleth maps in Figure 4. The exponentiated posterior estimates are interpreted as multiplicative factors on the hazard rate. Thus, negative posterior estimates imply a reduction in the hazard rate of fires, or analogously, an increase in the survival probability. The choropleth maps apply the brightest red colour of the heat palette to compartments with the largest negative posterior estimates to imply greater fire danger through to the palest yellow being applied to compartments with the largest positive posterior estimates to imply the least fire danger. The visualization of broad spatial patterns is achieved by using legend interval lengths which represent standard deviations from the mean of the random effect estimates. These choropleth maps suggest that there is a region in northwestern Ontario (FMC 15) where fires are more likely to experience shorter control times and that people-caused fires the extreme southeast (FMCs 1 and 2) also are more likely to experience shorter control times when compared to similar fires elsewhere in the study area. We return to this point in our discussion where we postulate the potential sources that may be contributing to this reduced risk.

4.2. Assessing Significance and Assumptions

Following Therneau and Grambsch (2000), we use a profile likelihood-based confidence interval as a preliminary assessment of the significance of the frailty terms in our models. At the 95% confidence level this interval is represented by the intersections of the likelihood ratio (LR) test statistics for a sequence of random effect variances with a horizontal line at the critical value of a chi-squared test on 1 degree of freedom. The resulting intervals for lightning and people-caused fires are (0.0137, 0.0733) and (0.0087, 0.0480), respectively. These intervals suggest that the frailty terms of the models are significant in terms of this preliminary assessment since they are relatively narrow. Therneau and Grambsch (2000) suggest that the significance of the frailty term is more strongly evaluated by an LR test comparing the integrated-likelihood

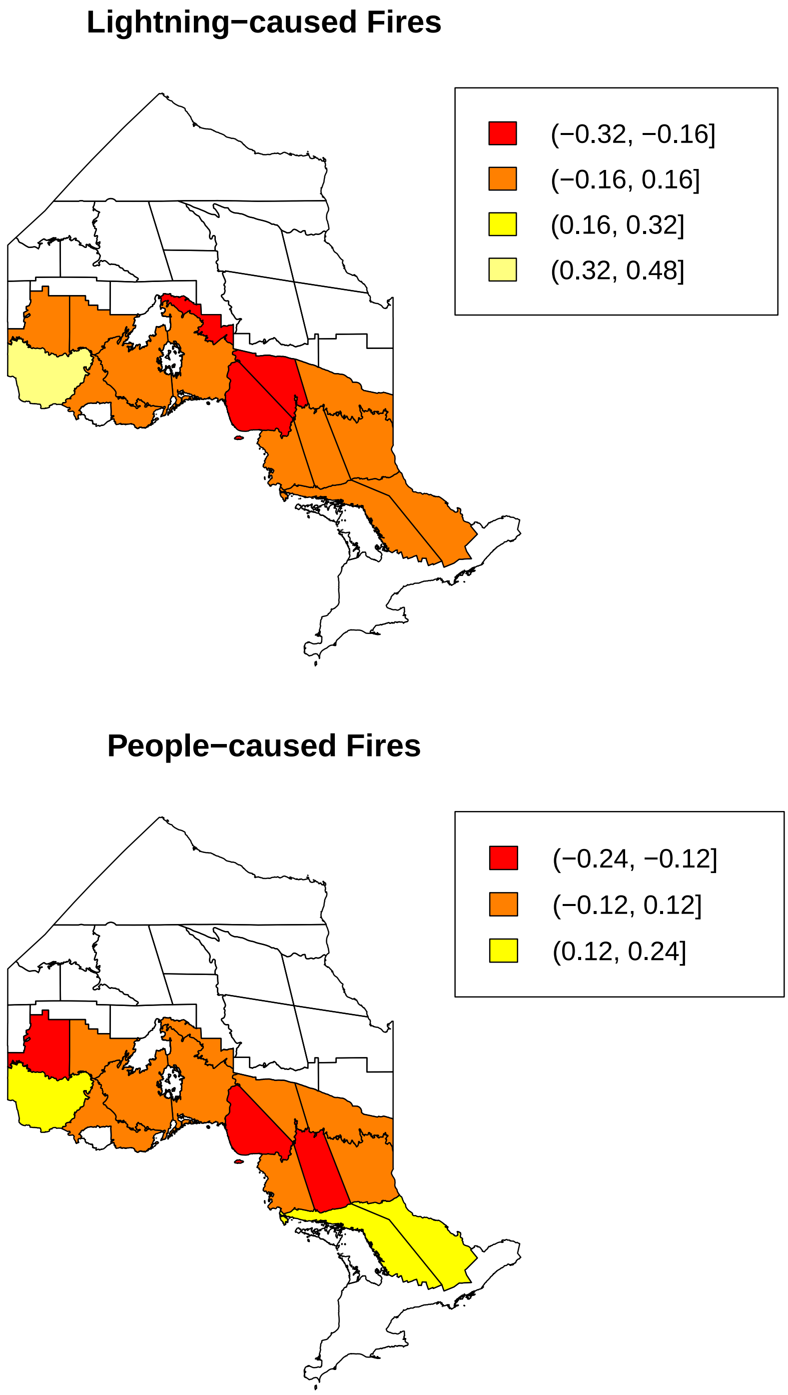

Figure 4: Choropleth maps of lightning (top panel) and people-caused (bottom panel) fires where each FMC in the Intensive Fire Management Zone (the study area) is assigned a heat map colour based on its estimated frailty term. For comparable fires across different FMCs, namely for fires where all other predictors are held constant, FMCs with negative frailties have fires with longer control times and FMCs with positive frailties have fires with shorter control times. Since gaussian frailties were employed, the colour pallettes use interval lengths that are equal to the standard deviations of the random effects for a given model, except for the interval that contains 0 (orange) which has a width equal to two standard deviations centred at zero.

of the frailty model, where the frailty terms are integrated out of the likelihood, to the Cox PH fitted likelihood without a frailty term. Based on this comparison, both the lightning and people-caused frailty models are significant improvements over the respective proportional hazards models without frailty terms. However, it is of note that this Chi-squared test is conservative since the frailty terms, e^{ω}, are constrained to be non-negative.

The suitability of using normally distributed frailty terms is verified by fitting Cox PH models with additional fixed effects for the FMCs. FMC 15, the compartment in the western region which experiences the largest number of lightning-caused fires, was chosen as the baseline compartment. To make the random effects from the frailty models comparable to the fixed effects, FMC 15's posterior estimate is subtracted from the other posterior estimates such that the values are representative of the multiplicative difference on the hazard rate of fires in FMC 15. The differences between these fixed and random effects are displayed in the choropleth maps in Figure 5. These differences are small, suggesting that the use of normally distributed frailty terms is reasonable.

5. Discussion

In this paper, the results of Morin *et al.* (2015), where the control time of forest fires was modeled using survival analysis methods, were extended to explore for spatial trends. This was achieved by incorporating random effects in the Cox PH models which improved the fits due to the presence of correlation between the control times of fires within a set of FMCs which formed a partition of the former Intensive Fire Management Zone of Ontario's fire region. The PH shared frailty models were each fit with a zero-mean normally distributed random effect term which represented the FMCs. These frailty terms were found to be significant by creating profile likelihood-based confidence intervals and by performing LR tests which compared the integrated-likelihoods of the frailty models and the likelihoods of the models without frailty terms.

The normal frailty assumption was verified by re-creating these choropleth maps using fixed effects for each FMC. The fixed effect parameter estimates were similar to the posterior random effect estimates which suggests that this assumption is reasonable. In the future, these types of frailty models could be extended to a region with more compartments such as the entire Province of Ontario.

The parameter estimates associated with the fixed effects were negative, indicating increases in the survival probability. This result is similar to that of the aspatial model in Morin *et al.* (2015) in which it was discussed that these relations are consistent with the structure of the fire weather variables (increasing with greater fire risk), as well as with the fire management beliefs that it is important to minimize initial attack response time and that larger fires are more diffcult to bring under control. The initial attack time of day effect also suggested that fires attacked later in the day tend to have greater survival probabilities, which is likely attributed in part to the overnight suspension of suppression action.

Choropleth maps were created using the posterior estimates of the random effects from each FMC. The map from the model of lightning-caused fires displayed more prominent patterns, resulting from a larger estimate for the variance of the random effect, than the map from the model of people-caused fires; in the latter, none of the compartments had posterior estimates which where greater than two standard deviations from the mean. While the random effect

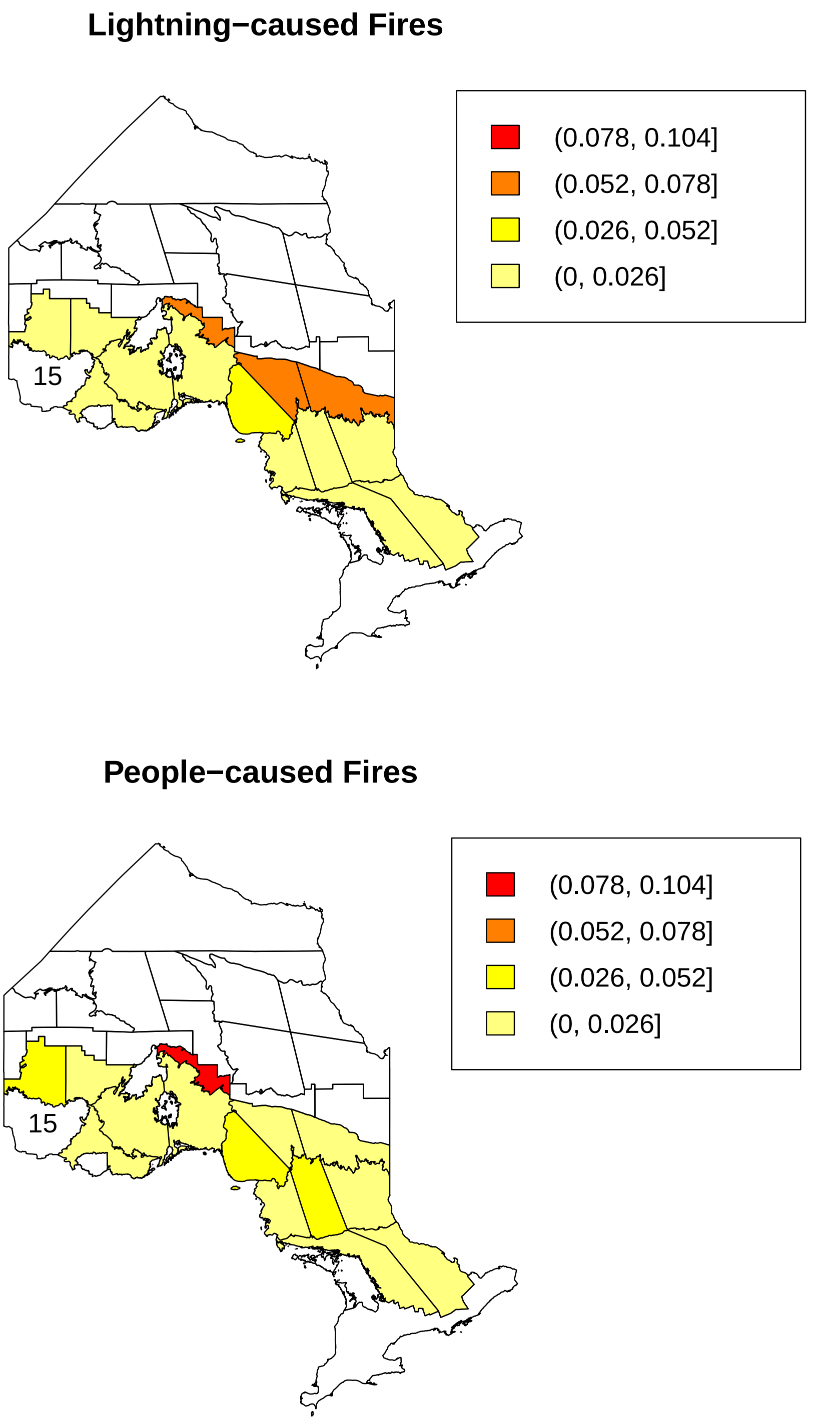

Figure 5: The differences between the parameter estimates of the compartment-specific fixed effects and the posterior estimates, in reference to FMC 15, from the frailty model of lightning (top panel) and people-caused (bottom panel) fires.

terms improved the fits of both models, possibly accounting for regional differences in land-use in the model of people-caused fires, the analysis of spatial patterns in the case of lightning-caused fires is of particular importance as lightning strikes tend to arrive in spatio-temporal clusters from passing storm systems (Woolford and Braun 2007). The choropleth maps of lightning-caused and people-caused fires illustrated that FMC 15 in the western region of the Intensive Fire Management Zone seems to be more likely to have fires with shorter control times than the average across the entire study region. Note that this compartment has both a number of small and medium sized communities (e.g., Kenora, Minaki, Sioux Narrows), and a number of indigenous communities (e.g., Naotkamegwanning First Nation, Obashkaandagaang First Nation), as well as a high concentration of vacation homes on and near Lake of the Woods. Recognition of theses values at risk coupled with the fact that Kenora District experienced high fire activity in the past have contributed to the Ontario Ministry of Natural Resources & Forestry deploying their initial attack resources such that they can quickly respond to fires that are reported in FMC 15. FMC 15's initial attack response time is also influenced by the fact that the adjacent province of Manitoba's wildfire management program can and does, from time to time, dispatch their airtankers to carry out "quick strike" initial attack on fires in Kenora district.

Fire management agencies are responsible for making important decisions which include strategic protection planning and fire suppression resource allocation. Some of these decisions, such as the daily deployment and prioritization of fires when heavy fire loads are being experienced, are critical to the protection of people, property and forest resources. The type of modelling presented in this paper could provide fire managers and planners additional information to consider when making such decisions. The choropleth maps presented in this study could be used to identify and classify areas of high or low risk in terms of the control time of forest fires after accounting for fire weather and fire management variables. This information could be used to inform fire management strategic planning processes such as strategic resource acquisition and home basing (placement of crews and aircraft), as well as regional targeting of programs aimed at raising public awareness about fire prevention. Choropleth maps of posterior estimates of frailty terms could be used to monitor the spatial "noise" in control time. For example, a historical period could be used to determine a baseline and posterior estimates of the frailty terms for the same modelling framework fit to more recent data, such as the current fire season or recent fire season(s), could be compared to their corresponding baselines. In this sense, the choropleth maps could also be used to monitor and revise fire management plans over time.

Our work could be extended in future related studies. For example, the control time analyzed in this paper was defined as the time period from the onset of initial attack to the fire being declared under control. A similar exploration of spatial patterns of this control time could be applied to alternative time periods including using the time of ignition or report of the fire as the lower-bound of the time period. It is of note however, that the time of ignition is often estimated and using this lower-bound would therefore require that left-censoring be accounted when fitting models to such data. Another potential future avenue of study would be to extend the framework to account for a sequence of time-dependent fire weather covariates that are observed over the entire control time of each fire, rather than individual fire-weather covariates observed at a fixed point in time as we employed in our modelling herein.

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